Using the Hysteresis Loop to Study a Single-Phase Transformer Working in AC-Switching Regime

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***Abstract*—A model to generate the hysteresis loop starting from the fundamental magnetisation curve is proposed in this paper. This was imposed by the necessity to improve the results returned by authors' previous studies about the periodical ac-switching mode of single-phase transformers. The hysteresis loop generation and tracing procedures are presented in the paper, the last one dealing with a transformer working in ac-switching mode, RL and RC loaded (with current or voltage rectifiers in secondary winding). Comparisons between experimentally and theoretically determined primary current waveforms are included in the paper, to prove the correctness of the proposed hysteresis loop model. MathCAD 6.0 was used as mathematical support.**

***Index Terms*—hysteresis loop, magnetisation curve, periodical ac-switching , single-phase transformer, voltage rectifiers.**

I. INTRODUCTION

The designing process of the single-phase transformers working in ac-switching regime imposes to determine their main parameters. A common practice is to obtain primary and secondary volt-amps, S_1 and S_2. These quantities are influenced by the rectifier and load circuits characteristics.

To determine the volt-amps of the transformer operating in these conditions, i.e. ac-switching mode, (this one being the main data to design whatever transformer), many authors have proposed a multiplying coefficient, k_T; $k_T > 1$, which is applied to the load active power and is tabled for various kinds of load circuits. The authors' intention is to make a contribution to optimization of this coefficient.

The first step in this direction has to be a study of transformers ac-switching process.

In ac-switching regime, the operation of the transformers differs from the "stationary" (no switched) one, because it implies non-symmetrical and no-sinusoidal currents and consecutively dc component of the magnetic field.

Owing this, a premagnetization in the core will be achieved, so the transformer may work in the saturation zone of the magnetisation curve.

Regarding these reasons, the paper will present only the operation of the transformer with a single-pulse rectifier in the secondary coil, which the worst case concerning the currents waveforms consists in.

The circuit diagram is shown in Figure 1. "V" was considered a ideal thyristor-type switch.

Previous studies were performed using several models for the transformer:

- The equivalent T series diagram [1-3];
- Transformer with the core characterized by a linear magnetization curve [4-7];

Transformer with the core characterized by a non-linear magnetization curve, without hysteresis loop [8,9].

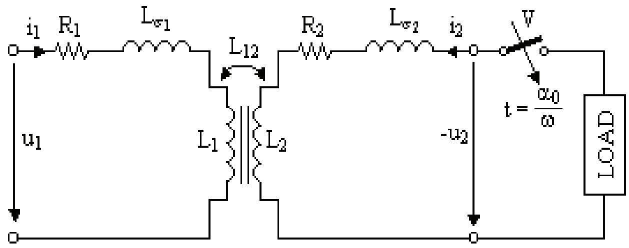

Figure 1. A single-phse transformer in ac-switching mode

Numerical methods are requested by the impossibility of writing analytical expressions of the signals characterizing the circuit operation (transcendental equations should be solved to determine the duty cycles durations). This is the way to obtain analyzed quantities numerical values (both in transient or steady state). MathCAD functions were created in this purpose.

Waveforms of the quantities characterizing the ac-switching of the transformers inner rectifiers (primary current specially) resulted behind these studies.

As validation criteria for the transformer model, comparison between theoretical and experimentally determined waveforms (shapes and magnitudes) was proposed.

Owing to the (relative) error level between theoretical and experimental waveforms returned by all of the above-mentioned models (fairly accurate waveforms being returned by [8,9] and the worst by [1-3]), the natural sequel is to take into account the hysteresis loses. This is the subject discussed in this paper.

II. GENERATING THE HYSTERESIS LOOP

The procedure developed for generating the hysteresis loop will be shortly presented. Its input data is the fundamental magnetization curve of the core material, experimentally determined and piecewise linearized.

The hysteresis loop width is assumed to be proportional with the magnetic induction corresponding to the turning point (P, see Figure 2; the sense of tracing the curve is

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changing).

It results a narrow hysteresis loop corresponding to the quasi-linear zone and a broad one for the non-linear (including saturation) zones.

The hysteresis loop is assumed as curves parallel with the fundamental one, the distance between them being in concordance with the above-mentioned (the red curve in Figure 2, then the blue one).

An intermediate line (PP_1 or QQ_1 in Figure 2) links these curves, characterised by a slope proportional with the magnetic permeability in the turning.

Figure 2 shows how this algorithm works: the point will trace the curve OP, then the red curve is generated, it is computed the point P_1 (or P_2), and the portion PP_1Q is traced. Here is generated a new curve (the blue one) and the portion QQ_1(3) will be traced (in ascendant sense) and so on.

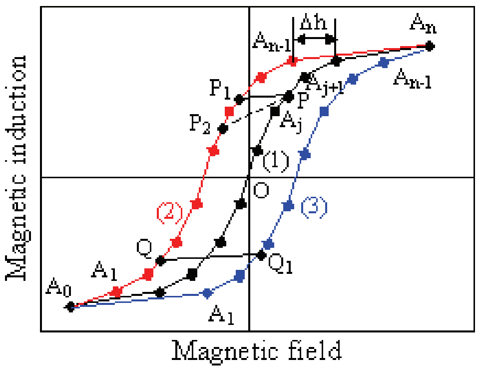

Figure 2. Hysteresis loop modelling

III. ROUTINES

A. The Main Routine

The main routine that controls the whole calculus process is shown in Figure 3.

Its input data are the following:

α_c: The angle corresponding to the moment when the circuit biased (by the sinusoidal voltage u_1, Figure 1);

Φ : A vector containing the load circuit phase-shifts;

A0 : A vector containing the firing angles of the thyristor;

M: A matrix containing the fundamental magnetization curve, previously experimentally determined and linearized;

Pf: The number of ac-cycles whom the study is fulfilled.

At the beginning of the calculus process, the electric quantities are initialized. The appropriate position of the (H, B) point on the magnetization is deriving in this way. These are the main tasks of the section Initializations.

The main routine is calling three subroutines:

NextStep, which is calculating the moment when the point on the magnetization curve reaches either the end of the actual portion of the (linearized) magnetization curve, or a extreme point of the magnetic field time variation;

Eval, which is evaluating the electric quantities values in the moment returned by NextStep and decides the state of the thyristor (ON or OFF);

WorkCurve, which generates a new magnetization curve, implementing the algorithm described in previous section.

This procedure is called when changing the tracing sense on the curve, i.e. when the magnetic field time variation reaches an extreme point.

Data returned by these three procedures are processed by the section Updates, fixing new values of their arguments.

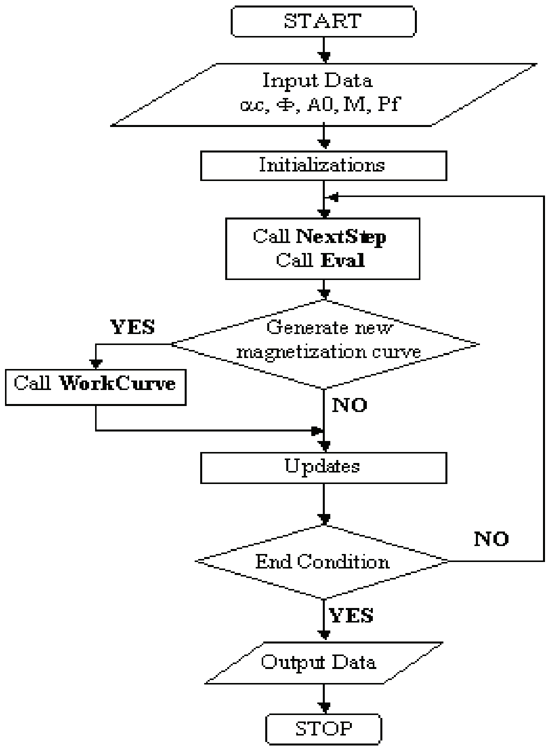

Figure 3. The main routine

The calculus process reaches its end when the **End Condition** (the ac-cycle index becomes equal with Pf) is TRUE.

In this moment the main routine returns the Output Data, which mainly are the initial values of the electric quantities at the beginning of a ac-cycle and the magnetization curve corresponding to the steady state regime. These are necessary to determine the interest signals waveforms

B. The Subroutine tracing the Magnetisation Curve

This one is called **NextStep** and is shown in Figure 4.

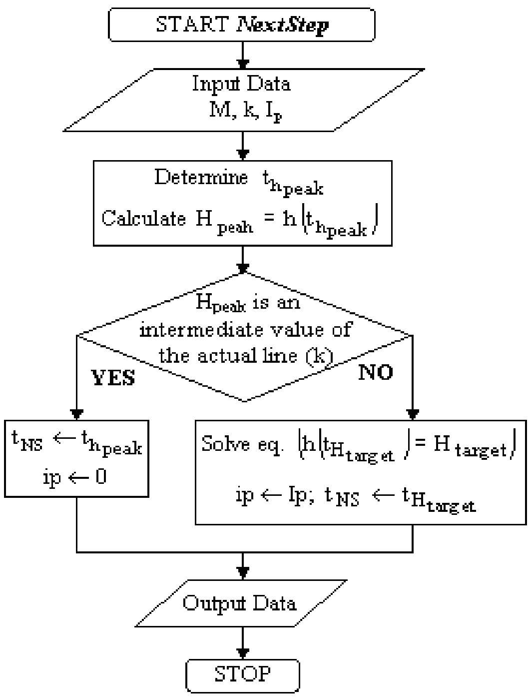

Figure 4. The Subroutine NextStep

Its input data are mainly the following:

M, the actual magnetization curve; it was previously linearized and its parameters are filling a matrix;

k, the line index of the (H, B) point position on the magnetization curve;

Ip, a variable characterizing the tracing sense on the curve:

$Ip = 1$ when tracing the curve in ascendant sense;

$Ip = -1$ when tracing the curve in descendant sense;

$Ip = 0$ when staying on the curve (in peak points).

The subroutine works in a very simple way: first, the moment $t_{h_{peak}}$ (when the time variation of magnetic field, h(t), reaches the peak point corresponding to the actual line of the magnetization curve), and the value of this peak, H_{peak}, are determined.

If H_{peak} is an intermediate value on the actual line, Fig 5a, it comes out that the peak point will be reached by h(t) and this situation will be announced by the zero value of **ip**.

Otherwise, the target point, H_{target}, (which is one of the actual line ends, Figure 5b) will be reached, in a moment resulting by solving the equation shown in Figure 4; in this case, the tracing sense doesn't change, so the variable **ip** will keep the **Ip** value (1 or -1).

The output data of this procedure are mainly the moment $\mathbf{t_{NS}}$ and the value of **ip**. It could be worth to point out that the value of **ip** will be used by the main routine to call the procedure generating the new magnetization curve

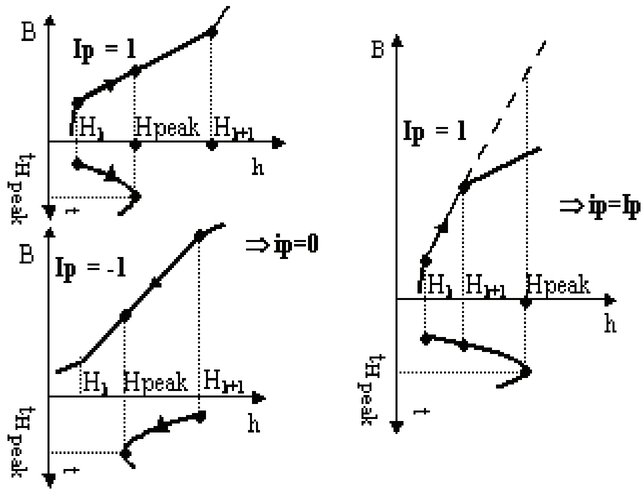

Figure 5. Tracing the magnetization curve

C. The Subroutine Eval

The task of this procedure is to evaluate the electric quantities in the time-moments returned by the function NextStep and to fix the state (ON or OFF) of the switch "V". Figure 6 shows this subroutine, whose (principal) input data are:

- **A0**, a vector containing the firing angles values, α_0;
- **NextStep** output data, meaning by this tNS, mentioned in subsection III. B.

The main deciding criteria in this subroutine are the secondary current value, i2(tNS), and the relation between tNS and $t\alpha_0$ (which is corresponding to α_0 value). As Figure 6 shows, 4 cases are possible:

- $i_2(t_{NS}) < 0$, i.e. the switch "V" is in ON state (according to thee senses in Figure 1). Eval will return t_{NS} and the switch state unchanged;
- $i_2(t_{NS}) = 0$, i.e. switch "V" is in OFF state, and $t_{NS} < t\alpha_0$. Eval will return t_{NS} and the switch state unchanged;
- $i_2(t_{NS}) > 0$, the switch being in ON state. Eval must find the switch turn-off moment, $t\beta$, changing in the same time the switch state in OFF;
- $i_2(t_{NS}) = 0$, i.e. the switch "V" is in OFF state, and $t_{NS} \geq t\alpha_0$. Eval will return $t_E = t\alpha_0$, changing in the same time the switch state in ON.

The main output data are the one already mentioned; information about changing or not the value t_{NS} are included in output data too.

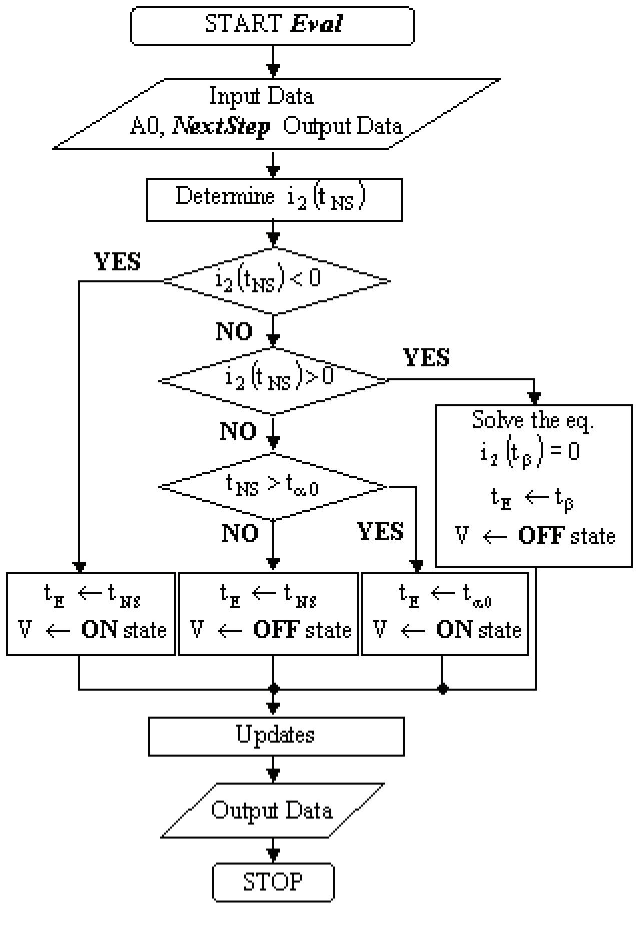

Figure 6. The Subroutine Eval

D. The Subroutine WorkCurve

This procedure generates a new magnetization curve when necessary, implementing the algorithm described in section II. It is called by the main routine when the magnetic field time variation, h(t) reaches peak points, Hpeak (maxims or minims).

As Figure 7 shows, its input data are the following:

- WL, the "Walking Line", i.e. the fundamental magnetization curve from whom the convex portion in origin zone was linearized;
- B1, the linearization of WL;
- iP, a variable characterizing the tracing sense on the curve. It is not the same with Ip mentioned in subsection

III.B, because it has only two values, 1 or -1.

- **Δh** , the hysteresis loop width (see Figure 2);

$$\Delta h = \frac{B_{peak}}{B_{sat}} \cdot H_c \tag{1}$$

- where B_{peak} is the magnetic induction corresponding to H_{peak}, B_{sat} is the saturation induction and H_c is the coercive force;
- **h** $= H_{peak}$;
- **nμ** , a coefficient applied to the actual magnetic permeability, where

$$0 < n\mu < 1; \tag{2}$$

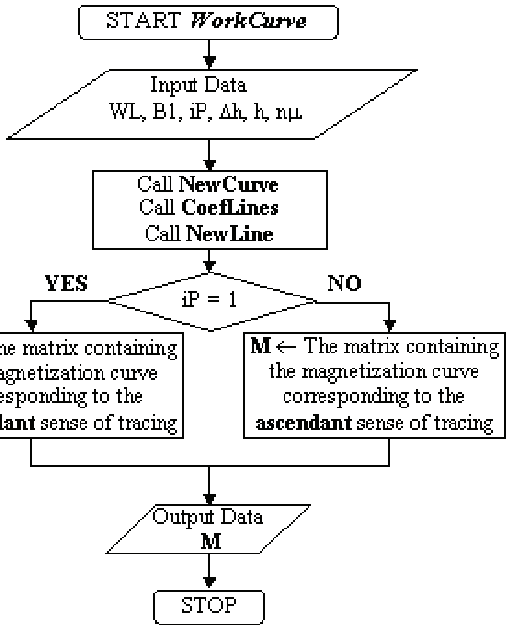

Figure 7. The Subroutine WorkCurve

WorkCurve calls three other subroutines:

- **NewCurve** is generating a curve parallel with WL, in the left or right side, at the distance Δh ;
- **CoefLines** generates the lines coefficients (slope and intercept) for linearizing a tabled function;
- **NewLine** links the actual magnetization curve to the curve generated by NewCurve; it is a line whose slope is given by

$$\mu_{link} = \mu_{actual} \cdot n\mu \tag{3}$$

Because of (2), it results

$$\mu_{link} < \mu_{actual} \tag{4}$$

It must be clearly pointed out that the lines parameters returned by CoefLines applied to the magnetization curve have dimensions: the slope is the (dynamic) magnetic permeability and the intercept is a magnetic induction. In this context, μ_{actual} in (3) is the slope of the line on which the peak point (H_{peak}, B_{peak}) was reached.

The **Output Data** consists in a matrix filled with the new magnetization curve linearization. It starts from the (turning) point (H_{peak}, B_{peak}). The structure of this matrix depends of the sense it will be traced: the linking line must be its end if the tracing sense will be descendant, or its beginning if the tracing sense will be ascendant. The deciding criterion in this matter is the actual value of **iP**, as Figure 7 shows. All the routines presented in this section (excepting WorkCurve) optimise the ones from [10], by a significant decrease of the necessary computing time.

The main routine was computed for a big enough number of ac-cycles, so the transformer reached its steady state. Once having a linearized magnetisation curve, the transformer will be a linear one on each line of the curve, so the equations are the same with ones included in [8] and [9].

The quantities characterizing the transformer (resistors, inductivities, number of turns, geometrical dimensions, fundamental magnetization curve) were previously experimentally determined, and some values of the load circuit (RC or RL) and the SCR firing angles that are the input data for this study are the same with the ones used in the measuring circuits.

IV. WAVEFORMS

The main routine presented in section III.A returns the signals initial values corresponding to the steady state. These are the input data of another function, whose task is to compute the signals values in a specified number of points/ac-cycle. The output data of this function can be used either to plot the waveforms, or for further studies, such Fourier analyse.

Magnetic permeability variation along an ac-cycle will be the first waveforms presented in this paper.

The magnetic permeability variation (see Figure 8) offers a possibility to “visualize” the hysteresis loop of the magnetization curve.

The magnetic permeability variation evidences:

- The magnetisation curve is piecewise-linearized, each of its lines being characterised by a magnetic permeability (the plots are “stair-type” ones);
- The hysteresis loop is influenced by the load (phase-shifts) and firing angles;
- The transformer reached its steady state, because the initial and final values of the magnetic permeability are the same.
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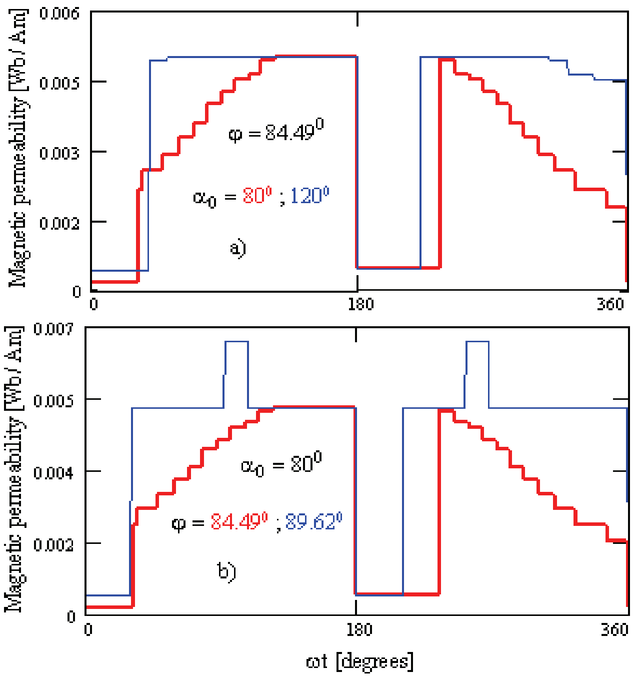

Figure 8. Magnetic permeability variation along an ac-cycle corresponding to RC load, for the same load, firing angle variable (a) and the same firing angle, load variable (b)

Figure 9 and 10 are showing the magnetic field and magnetic induction variations along an ac-cycle, $[\omega t \in (\alpha_0, 2\pi + \alpha_0)]$. These variations are presented as functions of firing angle α_0, and load phase-shift φ.

The following observations are coming out by correlating the above-mentioned plots:

- As load leading character increases, the magnetic field waveform is less deformed (comparing it to a sin wave). As a consequence, the magnetic quantities dc components are decreasing in this case.
- The dc component low level evidences a narrower hysteresis loop (i.e. lower losses), owing this to the operation on the "quasi-linear" zone of the magnetization curve.
- The magnetic induction plot is a sin wave, as it does in the non-switched case.

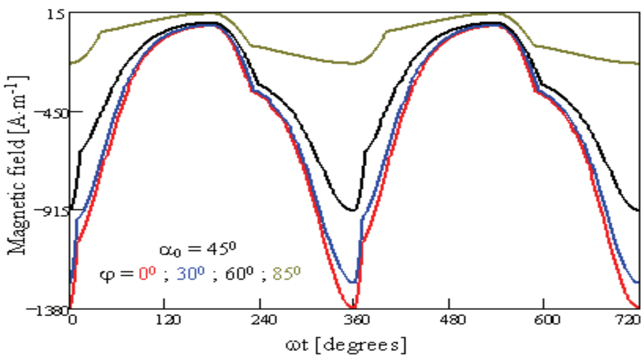

Figure 9. Magnetic field variation along an ac-cycle corresponding to RL load, for the same firing angle, load variable

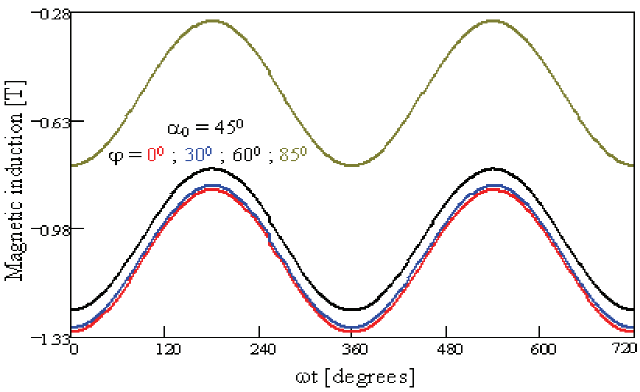

Figure 10. Magnetic induction variation along an ac-cycle corresponding to RL load, for the same firing angle, load variable

Either in [4-7] or in [8-10], the load current theoretical waveform is satisfactory enough if it is compared with an experimentally determined one.

This is the reason of non-introducing in this paper such waveforms, but only the primary current ones, which the main problem consists in. Some of the obtained waveforms are inserted in Figure 11 and 12, as comparisons between theoretical and experimentally determined ones

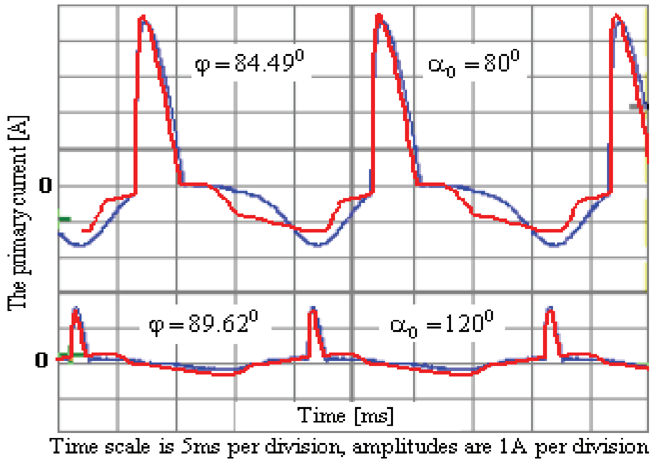

Figure 11. Comparison between the theoretical and experimentally determined primary currents for RC loaded transformer

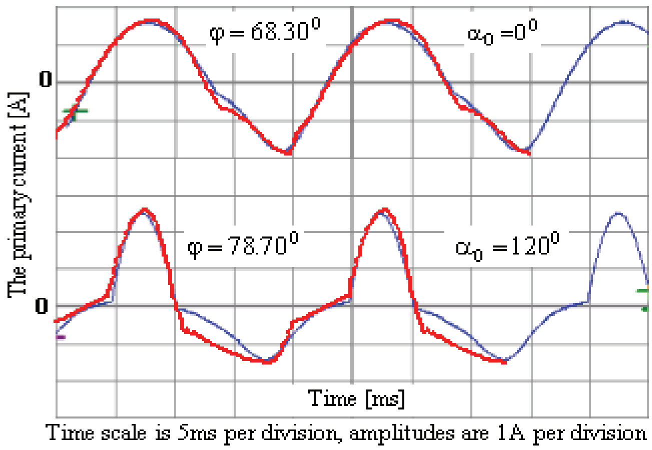

Figure12. Comparison between the theoretical and experimentally determined primary currents for RL loaded transformer

The experimental waveforms were obtained using a digital oscilloscope Hameg 305 and were transferred to a computer with the help of the data acquisition software SP 107 – V1.92. Figure 13 presents a capture of the computer display (the primary current waveform on the oscilloscope).

V. CONCLUSION

Because the hysteresis loop (in steady state) is depending on load and firing angles values, the hysteresis losses can be represented as a function of these variables. This observation can be used for designing transformers working in ac-switching regime.

The experiments prove the accuracy of our calculus. Comparative with [9] and [10], the primary current present a significantly lower error (the maximum one is about 5%), regarding the turn-off angles and (specially) amplitudes.

Certain improvements in the theoretical waveform shape can be observed as well. For instance, in the waveforms shown in Figure 11 and 12, it can be observed a pretty good approach between the two waveforms in the second part of the turn-off interval.

But it still remains a problem: in RL load case (Figure 12), the primary current values in the switch turn-off moment do not match at all with the experimentally determined ones.

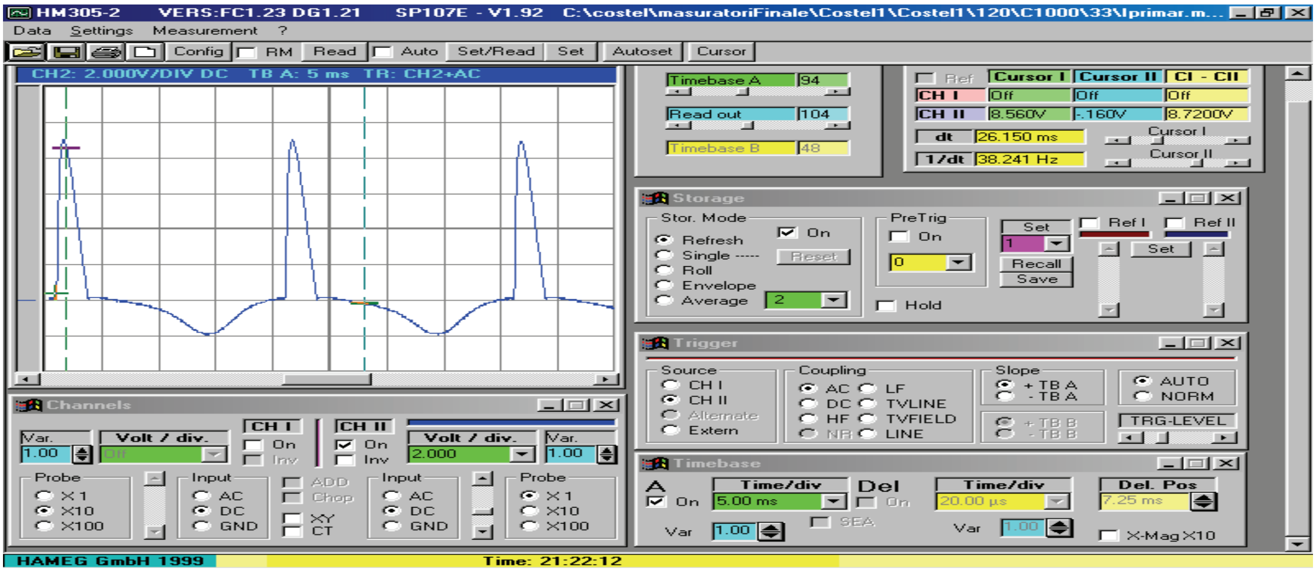

Fig. 13. Data acquisition using SP 107 – V1.92

It is not an easy problem, because it has many directions to face, such as:

- To not consider the switch ideal anymore. (In this direction, only a non-null value of the holding current, I_H was considered in the RL load case).
- To find another models of the magnetisation curve, instead the piecewise linearized one.

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