# Wireless System for Remote Tilt Measurement in Monitoring and Control Applications 

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#### Abstract

The wireless system for remote measurement of tilt measures the tilt angles of an object with respect to the local $g$-vector and communicates the measurements toward a mobile device able to display them or to transfer them further to a PC or to a PDA. A low cost implementation solution is presented, with references to both the hardware platform and to the main ideas behind the algorithms present in the software. Higher versatility is achieved through avoidance of the need to calibrate the sensor to the local value of the $g$ acceleration, using algorithms in which this value does not appear in the computation process.


Index Terms-Acceleration measurement, Intelligent sensors, Microcontrollers, Wireless communication

## I. INTRODUCTION

The wireless system for tilt measurement is meant for measuring the tilt of an object relative to the vertical direction (based on the direction of the gravitational force of the Earth). The measurement of the object's tilt can be done remotely (remote measurement) by using wireless communication. An autonomous device is built around a low-cost hardware platform, targeting to work with lowpower consumption, but without compromising the measurement accuracy or the sampling frequency.

While the applications of the system are numerous, ranging from monitoring the angles of large construction structures (bridges, large pipes, etc.) or usage for precise positioning of metallic structures to usage as input sensors for various monitoring and control systems (industrial, medical, agricultural, etc.), the proposed approach is paying particular attention to the setup of the vision based monitoring and control systems (see, for example, the application described in [4],[9]).

For a class of vision based control systems, the problems related to the calibration of these systems can be approached in a more robust way when the information about the relative placement of the cameras or of the calibration objects is available. Knowing the angles between the camera and the vertical direction or the angles between the calibration pattern and the vertical direction can be, for example, useful for the computation of the extrinsic parameters of the cameras.
Due to the fact that we are targeting such an application area, requirements such as small footprint, low weight, and reducing as much as possible the cabling through the use of wireless connections are very natural.
The problems that the proposed system is solving are the following:
A. The measurement of the gravitational acceleration
components on three orthogonal directions, by using a 3D accelerometer;
B. The conversion of these acceleration values into angle values relative to the direction and the sense of $g$ vector of the gravitational acceleration based on a algorithm implemented on a low-cost microcontroller;
C. Transmission of the measured values over a radio interface to a mobile acquisition unit which allows displaying of the measured values and transferring them to computer units such as a PC or PDA.

## II. ARCHITECTURE OF THE SENSOR DEVICE

The proposed measurement system uses a Sensor device (see Figure 1) built around a measurement and control module (Microcontroller) which is managing a radio interface ( $R F$ Transceiver), an $A / D$ Converter, a power management module ( $M A$ ) and a rechargeable battery (ACC).


Figure 1. Architecture of the sensor device.
Due to the availability of 2 D accelerometers with a higher sensitivity than that of the 3D accelerometers, a 3D acceleration sensor was implemented by means of two 2D sensors having one of their axes aligned, in order to form a common third axis.
The information received as three voltages ( $\mathrm{aX}, \mathrm{aY}, \mathrm{aZ}$ ) from the outputs of the 3D accelerometer is converted into numerical values ( $\mathrm{nX}, \mathrm{nY}, \mathrm{nZ}$ ) through the $\mathrm{A} / \mathrm{D}$ converter, transformed into three angular values through the algorithm implemented on the microcontroller unit and streamed over the RF transceiver radio interface when, over the same interface, a query message is received from a Acquisition Unit type of device.

If an algorithm based on the ratio between aX and aY
components is used for computing a tilt angle ( $\alpha x y$ ), it is not necessary to calibrate the sensor to the local value of the $g$ acceleration, because this value does not appear in the computation.

## III. COMPUTATION OF THE ANGLE

The algorithm for computing the angle that is implemented on the microcontroller unit uses for the arctangent function an approximation under the form of ratio of two polynomials.

Combined with a method for reducing the current interval to an interval in which this ratio approximates well enough the arctangent function [1], this method provides quite some advantages as described in what follows.
Starting with the evaluation of the precision given by the series expansions of the arctangent and sine functions:
$\tan ^{-1} x=x-\frac{x^{3}}{3}+\frac{x^{5}}{5}-\frac{x^{7}}{7}+\frac{x^{9}}{9}-\frac{x^{11}}{11}+\ldots$ and
$\sin ^{-1} x=x-\frac{x^{3}}{3!}+\frac{x^{5}}{5!}-\frac{x^{7}}{7!}+\frac{x^{9}}{9!}-\frac{x^{11}}{11!}+\ldots$
(as indicated in the Table 1 for the value of $x=1.0$ ), there are some immediate remarks.

First of all, the sine approximation obtained truncating the series after the $3^{\text {rd }}$ or $4^{\text {th }}$ term $(n=3, n=4)$ is quite good, while the performance of the similarly truncated series for arctangent is very poor.

Secondly, even when retaining the first eight terms of the series, the approximation error of the arctangent is larger then 1.5 degrees (see line 8 in Table 1, with the error expressed in radians).

| TABLE 1 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $n$ | Sine |  | Arctangent |  |
|  | Sum | Error | Sum | Error |
| 1 | 1 | -0.15853 | 1 | -0.2146 |
| 2 | 0.833333333 | 0.008138 | 0.666666667 | 0.118731 |
| 3 | 0.841666667 | -0.0002 | 0.866666667 | -0.08127 |
| 4 | 0.841468254 | $2.73 \mathrm{e}-06$ | 0.723809524 | 0.061589 |
| 5 | 0.84147101 | $-2.5 \mathrm{e}-08$ | 0.834920635 | -0.04952 |
| 6 | 0.841470985 | $1.6 \mathrm{e}-10$ | 0.744011544 |  |
| 7 | 0.841470985 | $-7.6 \mathrm{e}-13$ | 0.820934621 | -0.031387 |
| 8 | 0.841470985 | $2.79 \mathrm{e}-15$ | 0.754267954 | 0.03113 |

This behavior of the power series, as well as previous experience with approximations based on rational fractions [7], led the authors to the search of an approximant in the $f(x)=P(x) / Q(x)$ form of a ratio of two polynomials: .

Such an approximation method gives the proposed device the advantage of the implementation on a low-cost microcontroller because it does not require high performance for numerical computations as implied by high order series approximations.

At the same time, if the degrees of the two polynomials are small enough, the algorithms of this type allow a significant reduction of the number of operations, compared to the ones based on series approximation.

This reduction of the number of arithmetic operations means a reduction of the processor cycles used for the computation so, in the end, it allows a significant decrease
in the power consumption, assuring a greater autonomy of the Sensor device.

Five approximations for the arctangent are indicated in Table 2 conform with [1], as well as their error in the point $x=1$.

| $f(x)=\frac{P(x)}{Q(x)}$ | Order | Fraction | Error |
| :---: | :--- | :--- | :---: |
| $x$ | 1 | $1 / 1$ | -0.214601836 |
| $\frac{3 x}{3+x^{2}}$ | 3 | $3 / 4$ | 0.035398163 |
| $\frac{x\left(15+4 x^{2}\right)}{15+9 x^{2}}$ | 5 | $\frac{19}{24}$ | -0.006268503 |
| $\frac{x\left(105+55 x^{2}\right)}{105+90 x^{2}+9 x^{4}}$ | 7 | $\frac{160}{204}$ | 0.001084437 |
| $\frac{x\left(945+735 x^{2}+64 x^{4}\right)}{945+1050 x^{2}+225 x^{4}}$ | 9 | $\frac{1744}{2220}$ | -0.000187422 |

The value $x=1$ is in fact corresponding to an angle value of $\pi / 4$. As suggested by figure 2 , if we denote by x and y the values of the acceleration components aX and aY , then the ratio of these values together with their sign allows an unambiguous computation of the corresponding angular value. It will be enough to exchange the values of $x$ and $y$ and to take the absolute value of the ratio in order to reduce the problem to a ratio in the range $[0,1]$, corresponding to angular values $[0, \pi / 4]$.


Figure 2. Relation between the tilt angle and the acceleration projections.
This is the reason for devoting further efforts to finding of a better approximation technique just for the interval of angular values $[0, \pi / 4]$.

It is well known that equation

$$
\tan ^{-1}(x)-\tan ^{-1}(y)=\tan ^{-1}\left(\frac{x-y}{1+x y}\right)
$$

is satisfied if and only if $\mathrm{xy}>-1$. (see, e.g., Formula 1.625.9, [5]).

Expressing the input value $x$, with the notations $z=\frac{x-k}{1+k x}, \tan a=z, \tan b=k$, we have:
$\tan ^{-1}(x)=\tan ^{-1}(k)+\tan ^{-1}\left(\frac{x-k}{1+k x}\right)=b+\tan ^{-1}(z)=a+b$
It can be observed that the value of interest, $\tan ^{-1}(x)$, can be computed with a simple addition:
$\tan ^{-1}(x)=a+b$,
where $a=\tan ^{-1}(z)$ and $b=\tan ^{-1}(k)$.
In other words, the problem of computing the arctangent $\tan ^{-1}(x)$ for a value $x$ expressed as the tangent of the sum of two angles, $x=\tan (a+b)$, is reduced to the problem of computing it for a smaller value $z$, and adding to it a constant $b=\tan ^{-1}(k)$ :
$\tan ^{-1}(x)=\tan ^{-1}(k)+\tan ^{-1}(z)$.
This technique is useful in shifting the input range into an interval in which the polynomial fraction approximant offers a better precision.
It is also possible to combine it with a partitioning of the input range $[0,1]$ into subintervals, and choice of an appropriate $b$ value for each subinterval.

## IV. ANALYSIS OF THE ERRORS

The levels of the errors introduced in the computation of the angle are investigated in what follows considering both errors generated by the approximant used and errors due to quantization of the original analog signal of the accelerometer.

Consider, first, the division of the $[0, \pi / 4]$ interval in four subintervals (the range reduction technique is applied using four different values of $b$ ).
Table 3 lists the errors (in degrees) obtained with the different order approximants at the ends of the $[0,1]$ input range.

Table 3

| Order | Error $\left[{ }^{\circ}\right]$ |  |
| :--- | :--- | :---: |
|  | $x=0$ | $x=1$ |
| 1 | 0 | 0.02166735463374 |
| 3 | 0.01903514269537 | 0.00964360439600 |
| 5 | 0.01898468506066 | 0.00966709156143 |
| 7 | 0.01898481413391 | 0.00966704729530 |
| 9 | 0.01898481380777 | 0.00966704737771 |



Figure 3. First order approximation error (4 subintervals).


Figure 4. First order approximation (4 subintervals).
Figures 3 and 4 indicate the results obtained with a first order approximant and with 4 subintervals compared with the Matlab arctan function. Better results are obtained with a third order approximant (see figure 5).

It is remarkable that even the first order approximant gives a error less the 0.05 degrees.


Figure 5. Third order approximation error (4 subintervals).

Table 4

| Order | Error $\left[{ }^{\circ}\right]$ |  |  |
| :--- | :--- | :---: | :---: |
|  | $x=0$ | $x=1$ |  |
| 1 | -0.14896902673401 | 0.14317363592741 |  |
| 3 | -0.0006434354608665902 | -0.00514533481799 |  |
| 5 | -0.00217193778396 | -0.00361694553563 |  |
| 7 | -0.00215670213858 | -0.00363217960611 |  |
| 9 | -0.00215685219707 | -0.00363202956754 |  |

The division of the $[0, \pi / 4]$ interval in just two equal subintervals (range reduction technique applied using two different values of $b$ ) is of interest due to the speed increase (just one comparison operation).

As indicated in figure 6, an error less than 0.2 degrees can be obtained with the first order approximant.

The third order approximant shows a much better performance, as indicated by figure 7 .

The precision can be further improved by choosing an optimal (not uniform) division of the interval.

Before going to look at the quantization errors, it is worthwhile to examine the specific intervals in which the numerical values ( $\mathrm{nX}, \mathrm{nY}, \mathrm{nZ}$ ) corresponding to the output voltages ( $\mathrm{aX}, \mathrm{aY}, \mathrm{aZ}$ ) of the 3D accelerometer are lying.

As it can be seen from the figure number 8, the positions of the $g$ vector draw a sphere in the coordinates system of the 3D sensor. For any such a position, one of the XOY, XOZ, YOZ surfaces has relative to $\vec{g}$ an angle lower than $\arccos (\sqrt{2 / 3})$ (Figure 9) so the amplitude of the signal applied to one of the two 2D sensors is greater than $\sqrt{2 / 3} \mathrm{~g}$.


Figure 6. First order approximation error (2 subintervals).


Figure 7. Third order approximation error (4 subintervals).


Figure 8. Possible positions of $\vec{g}$ vector in the coordinate system of the 3D sensor.

Under the conditions of 10bit quantization of the output signals of the accelerometers, this translates in our case to a variation of the $\mathrm{aX}, \mathrm{aY}, \mathrm{aZ}$ components in the range [418,512].

In order to establish the contribution of the quantization error in the angle computation based on arctangent, this error was analyzed for $\mathrm{aX}^{2}+\mathrm{a}^{2}=\mathrm{pXY} Y^{2}$, where pXY gets values in the above mentioned range.


Figure 9. Tridimensional distribution of the cosine of the minimum projection angle.


Figure 10. Maximum quantization error.

## V. THE WIRELESS SETUP

Due to the fact that the volume of data that will be sent toward the computer does not require a large bandwidth, a low band wireless communication system, for example, the WirelessUSB ${ }^{\text {TM }}$ LS [2], [3] can be used.

This is a low-cost system, which uses the 2.4 GHz band and which is composed out of a WirelessUSB LS bridge and at least one WirelessUSB LS HID (Human Interface Device) device.

The communication system WirelessUSB LS splits the communication channels either based on a code or on frequency. The 2.4 GHz band can be divided over 79 distinct sub-channels.

The acquisition and communication system based on WirelessUSB is depicted in Figure 11. The Sensor is connected to a WirelessUSB LS HID device. Through this, the information is sent toward the acquisition unit which is in turn connected to a WirelessUSB LS bridge. Finally the information reaches the PC/PDA through an USB port.


Figure 11. Aquisition and communication system.


Figure 12. Prototype device.

## VI. CONCLUSION

The implemented prototype device (Figure 12) allows the computation of the angles with a precision better than 0.1 degree, considering both the quantization error and the approximation error for the arctangent function using the first order approximation with four subintervals.

A low-cost, low pin count 8 bit microcontroller with an 8051 core and 8 Kbytes code memory was used. The 2D accelerometers are from ADXL family (Analog Devices). The prototype system was tested with both wired connection (RS232 or USB 2.0 [5]), and WirelessUSB wireless link toward a PC.

The prototype is able to do real-time measurements on tilting objects and to communicate the results of the measurement to other systems for decision and control. The architecture demonstrated with the application described could be easily followed or extended for implementing intelligent sensors for other measurands [8].

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