# Recent Research Progress in Fault Analysis of Complex Electric Power Systems

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Abstract—In this paper, we adopt a novel approach to the fault analysis of complex electric power systems. Electric power system is one of the most complex artificial systems in the world. Its safe, steady, economical and reliable operating plays a very important part in guaranteeing socioeconomic development, and even in safeguarding social stability. The complexity of electric power system is determined by its characteristics about constitution, configuration, operation, organization, etc. No matter if, we adopt new analytical methods or technical means, we must have a distinct recognition of electric power system itself and its complexity, and increase analysis continuously, operation and control level. In this paper, utilizing real-time measurements of phasor measurement unit, based on graph theory and multivariate statistical analysis theory, we are using mainly Breadth-first search, Depth-first search and cluster analysis. Then, we seek for the uniform laws of marked changes of electrical quantities. Then we can carry out fast and exact analysis of fault component. Finally, we can accomplish fault isolation. According to line fault and bus-bar fault (single-phase fault, phase-to-phase fault and three-phase fault) in complex electric power systems, we have carried out a great deal of simulation experiments and obtained ideal results. These researches have proven that the faults in complex electric power systems can be explored successfully by analysis and calculation based on graph theory and multivariate statistical analysis theory.

*Index Terms*—Complexity, Graph theory, Multivariate statistical analysis theory, Fault analysis, Electric power system.

# I. INTRODUCTION

The electric power system is one of the most complex artificial systems in this world and its safe, steady, economical and reliable operating plays a very important part in guaranteeing socioeconomic development, even in safeguarding social stability. In early 2008, the snow and ice disaster that occurred in south China had confirmed it again. The complexity of electric power system is determined by its characteristics: constitution, configuration, operation, organization, etc, that have caused many disastrous accidents, such as the large-scale blackout of America-Canada electric power system on August 14, 2003 and the large-scale blackout of Italy electric power system on September 28, 2003. To solve this complex and difficult problem, some methods and technologies reflecting modern science and technology level have been introduced, such as communication computer and technology, control technology, superconduct and new material technology, and so on. Obviously, no matter if we adopt, new analytical methods or technical means, we must have a distinct recognition of electric power system itself and its complexity, and continuously increase analysis, operation

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and control level. [1-3]

A fault is defined as a departure from an acceptable range of an observed variable or calculated parameter associated with systems. It may arise in the basic technological components or in its measurement and control instruments, and may represent performance deterioration, partial malfunctions or total breakdowns. Fault analysis implies the capability of determining, either actively or passively, whether a system is functioning as intended or as modeled. The goal of fault analysis is to ensure the success of the planned operations by recognizing anomalies of system behavior. A system with faults does not necessarily imply that the system is not functioning. Detecting a fault involves identifying a characteristic of the system, which, when a fault occurs, can be distinguished from other characteristics of the system. According to nonlinear complex systems, we have carried out large number of basic researches, [4-8]. In this paper, based on graph theory and multivariate statistical analysis theory, we will discuss the complexity of electric power systems.

The complexity of power network is mainly determined by the network structure, network vulnerability, cascading failure mechanism and so on. The fault in electric power system cannot be completely avoided. When electric power system operates from normal state to failure or abnormal, its electric quantities (current, voltage, angles, etc.) may change significantly. In our researches, after some accidents, utilizing real-time measurements of phasor measurement unit (PMU), [9-12], based on graph theory and multivariate statistical analysis theory, we are using mainly Breadth-first search (BFS), Depth-first search (DFS) and cluster analysis technology, [13-18], and we seek for the uniform laws of marked changes of electrical quantities. Then we can carry out fast and exact analysis of fault component. Finally, we can accomplish fault isolation.

## II. SEARCH PRINCIPLES IN GRAPH THEORY

Many real world situations can conveniently be described by means of a diagram consisting of a set of points together with lines joining certain pairs of these points. In mathematics and computer science, graph theory is the study of graphs: mathematical structures used to model conjugated relations between objects from a certain collection. A graph is an abstract notion of a set of nodes and connection relations among them, that is, a collection of vertices or nodes and a collection of edges that connect pairs of vertices. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to

## Advances in Electrical and Computer Engineering

another.

Applications of graph theory are primarily, but not exclusively, concerned with labeled graphs and various specializations of these. Structures that can be represented as graphs are ubiquitous, and many problems of practical interest can be represented by graphs. For example, in electric circuit theory, the Kirchhoff's voltage law and Kirchhoff's current law are only concerned with the structures and properties of the electric circuit. Then, any concrete electric circuit can be abstracted as a graph. [19] Here, let us give a simple electric circuit (See Fig. 1), and its structure can be expressed as a graph (See Fig. 2).

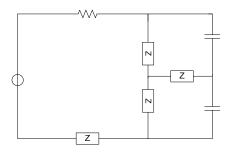


Figure 1. A simple electric circuit.

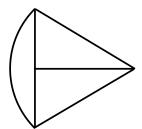


Figure 2. A graph based on the simple electric circuit.

Graph theory can be used to model many different physical and abstract systems such as transportation and communication networks, models for business administration, political science, psychology and so on. Efficient storage and algorithm design techniques based on the graph representation make it particularly useful for computer use. There are many algorithms that can be applied to solve different kinds of problems, such as Breadth-first search, Depth-first search, Bellman-Ford algorithm, Dijkstra's algorithm, Ford-Fulkerson algorithm, Kruskal's algorithm, the Nearest neighbor algorithm, Prim's algorithm, etc. Hereinto, Breadth-first search (BFS) is a graph search algorithm that begins at the root node and explores all the neighboring nodes. Then, for each of those nearest nodes, it explores their unexplored neighbor nodes, and so on, until it finds the goal.

BFS is an uninformed search method that aims to expand and examine all nodes of a graph or combinations of sequence by systematically searching through every solution. In other words, it exhaustively searches the entire graph or sequence without considering the goal until it finds it. From the standpoint of the algorithm, all child nodes obtained by expanding a node are added to a first-in, firstout (FIFO) queue. In typical implementations, nodes that have not yet been examined for their neighbors are placed in some container (such as a queue or linked list) called "open", and then, once examined, are placed in the container "closed" [20]. BFS can be used to solve many problems in graph theory, for example:

- Testing whether graph is connected, and finding all connected components in a graph;
- Computing spanning forest of a graph;
- Computing, for every graph vertex, a path with the minimum number of edges between start vertex and current vertex or reporting that no such path exists;
- Computing a cycle in graph or reporting that no such cycle exists.

The Depth-first search (DFS) is an algorithm for traversing or searching a tree, tree structure, or graph. One starts at the root and explores as far as possible along each branch before backtracking [21].

In formal way, DFS is an uninformed search that progresses by expanding the first child node of the search tree that appears and by going deeper and deeper until a goal node is found, or until it reaches a node which has no child node. Then the search backtracks, and it will return to the latest node that it has not finished exploring. The space complexity of DFS is much lower than BFS. It also lends itself much better to heuristic methods of choosing a likely looking branch. Time complexity of both algorithms is proportional to the number of vertices plus the number of edges in the graphs they traverse.

## III. CLUSTER ANALYSIS

Theories of classification come from philosophy, mathematics, statistics, psychology, computer science, linguistics, biology, medicine, and other areas. Cluster analysis can also be named classification, which is concerned with researching the relationships within a group of objects in order to establish whether the data can be summarized validly by a small number of clusters of similar objects. That is, cluster analysis encompasses the methods used to:

- (1) Identify the clusters in the original data;
- (2) Determine the cluster number in the original data;
- (3) Validate the clusters found in the original data.

Cluster analysis is commonly applied for statistical analyses of large amounts of experimental data exhibiting some kind of redundancy, which allows for compression of data to amount feasible for further exploration. This permits further mining of each cluster independently or, alternatively, constructing a high-level view of the data set by replacing each cluster with its best single representative. Cluster analysis is powerful in data analysis and has been applied successfully in various field researches. The effectiveness of a cluster approach depends on many choices. These include the choice of a cluster algorithm, an appropriate feature subspace, and a similarity metric defined over this subspace. In addition, cluster algorithms typically have a set of tunable parameters inherent to them that can heavily influence their performance. For example, many algorithms require the number of clusters desired, the maximum number of iterations, learning rate, its change schedule, etc. While some of these choices are obvious for simple artificial datasets. The most common clustering algorithm choices are hierarchical cluster analysis.

# Advances in Electrical and Computer Engineering

# Volume 10, Number 1, 2010

# IV. BFS AND DFS BASED FAULT ANALYSIS

Now let us consider IEEE9-Bus system. Fig. 3 presents the IEEE 9-Bus system electric diagram. In the structure of electric power system, Bus1 appears single-phase to ground fault. Through simulation experiments, using these actual measurement data of corresponding variables, we can carry through fault analysis of fault component and non-fault component.

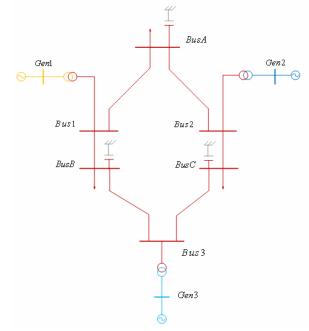


Figure 3. Electric diagram of IEEE 9-Bus system.

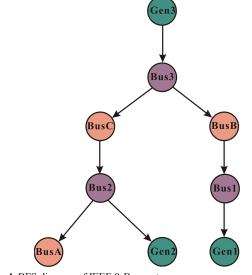


Figure 4. BFS diagram of IEEE 9-Bus system.

The adjacency matrix of IEEE9-Bus system can be expressed as follows,

	Bus1	Bus2	Bus3	BusA	BusB	BusC	Gen1	Gen2	Gen3	
Bus1	( 0	0	0	1	1	0	1	0	0)	
Bus2	0	0	0	1	0	1	0	1	0	
Bus3	0	0	0	0	1	1	0	0	1	
BusA	1	1	0	0	0	0	0	0	0	
BusB	1	0	1	0	0	0	0	0	0	
BusC	0	1	1	0	0	0	0	0	0	
Gen1	1	0	0	0	0	0	0	0	0	
Gen2	0	1	0	0	0	0	0	0	0	
Gen3	0	0	1	0	0	0	0	0	0 )	

By simulation experiments, we can get node phase voltage at  $T_{-1}$ ,  $T_0$  (Fault),  $T_1$ ,  $T_2$  and  $T_3$  five times (see Table I).

TABLE I. THE NODE PHASE VOLTAGE AT T-1, T0 (FAULT), T1, T2

Bus	<i>T</i> .1	T <sub>0</sub> (Fault)	$T_1$	$T_2$	$T_3$
Gen1	1.0100	0.7275	0.6924	0.6814	0.6747
Gen2	1.0100	0.8762	0.8476	0.8327	0.8134
Gen3	1.0100	0.8449	0.8071	0.7909	0.7710
Bus1	1.0388	0	0	0	0
Bus2	1.0430	0.7622	0.7350	0.7217	0.7049
Bus3	1.0534	0.7600	0.7275	0.7134	0.6960
BusA	1.0319	0.7540	0.7248	0.7114	0.6944
BusB	1.0222	0.2512	0.2404	0.2356	0.2294
BusC	1.0061	0.2470	0.2381	0.2336	0.2276

Fig. 4 is the BFS process of IEEE9-Bus system. In this diagram, Gen1 is the first generator node, it is also one of the terminals of BFS, and Bus1 is the only node that connects with it. Combining the information characters of electrical measurements that have marked changes, the difference of Bus1 and other Buses is distinct. At the beginning, Bus1 has been set as single-phase to ground, which is a typical bus-bar fault. In the final analysis, both these two aspects are consistent, and we can identify effectively fault location, based on BFS.

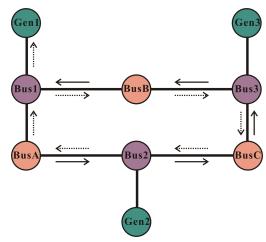


Figure 5. DFS diagram of IEEE 9-Bus system.

Dendrogram using Average Linkage (Between Groups)

Rescaled Distance Cluster Combine

CASE		0	5	10	15	20	25
Label	Num	+	+	+	+	+	+
BusB	8	-++					
BusC	9	-+ +					+
Bus1	4	+					
Bus2	5	-+					
Bus3	6	-+					
BusÅ	7	-+					Ι
Gen1	1	-+					+
Gen2	2	-+					
Gen3	3	-+					

Figure 6. The dendrogram of hierarchical cluster analysis based on node phase voltage.

Advances in Electrical and Computer Engineering

Volume 10, Number 1, 2010

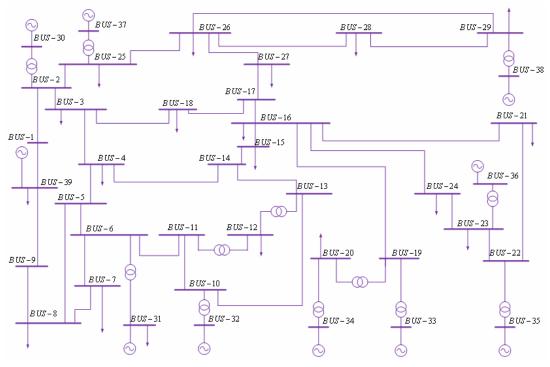


Figure 7. Electric diagram of IEEE 39-Bus system.

Fig. 5 is the DFS process of IEEE9-Bus system. In this diagram, the difference among Bus1 and other Buses is more distinct. Gen1 is the terminal of DFS, and Bus1 is the only node that connects with it. In the beginning, we have set Bus1 as single-phase to ground fault. Both of these two aspects are consistent. Therefore, we can also identify fault location effectively, based on DFS.

# V. FAULT ANALYSIS BASED ON HIERARCHICAL CLUSTER ANALYSIS

The hierarchical cluster analysis does not require to specify the desired number of clusters K, instead, affording a cluster dendrogram. In practice, the choice can be based on some domain specific and often have subjective components. There are three steps to hierarchical cluster analysis. First, we must identify an appropriate proximity measure. Second, we need to identify the appropriate cluster method for the data. Finally, an appropriate stopping criterion is needed to identify the number of clusters in the hierarchy. The distance or similarity metric used in cluster is crucial for the success of the cluster method. Euclidean distance and Pearson correlation are among the most frequently used.

Now let us continue to consider IEEE9-Bus system. According to the results of the simulation experiments (Table I), based on node phase voltage, we carry out hierarchical cluster analysis. Fig. 6 is the dendrogram of hierarchical cluster analysis based on node phase voltage.

Let us explain the entire process of cluster analysis in detail. The entire cluster analysis process is carried out according to the principle of similarity from high to low (distance from near to far). The order is:

Step 1: BusC is combined with BusB to form the new BusB;

Step 2: Bus3 is combined with Bus2 to form the new Bus2;

Step 3: BusA is combined with Bus2 to form the new Bus2; Step 4: Bus2 is combined with Gen1 to form the new Gen1; Step 5: Gen3 is combined with Gen2 to form the new Gen2; Step 6: Gen2 is combined with Gen1 to form the new Gen1; Step 7: BusB is combined with Bus1 to form the new Bus1; Step 8: Gen1 is combined with Bus1 to form the new Bus1.

From the entire hierarchical cluster process analysis, Busl has the lowest similarity to other nodes (the farthest distance to other nodes). It can also be found out easily, from Fig. 6, that Busl differs remarkably from other buses, and the fault characteristic is obvious. These results are identical to the fault location set in advance, so we can confirm exactly fault location by the hierarchical cluster analysis.

Let us further consider a more complex three-phase shortcircuit fault in IEEE39-Bus system. The IEEE39-Bus system is well known as 10-machine 39-bus New-England Power System, and Fig. 7 presents its electric diagram. In the structure of electricity grid, Bus-18 is just the fault location. According to the results of the simulation experiments, based on node positive sequence voltage, we carry out hierarchical cluster analysis. Fig. 8 is the dendrogram of hierarchical cluster analysis based on node positive sequence voltage.

According to the principle of similarity from high to low (distance from near to far), the order of the entire hierarchical cluster analysis process is:

- Step 1: Bus23 is combined with Bus22 to form the new Bus22;
- Step 2: Bus8 is combined with Bus7 to form the new Bus7;
- Step 3: Bus12 is combined with Bus5 to form the new Bus5;
- Step 4: Bus32 is combined with Bus31 to form the new Bus31;
- Step 5: Bus11 is combined with Bus10 to form the new Bus10;
- Step 6: Bus33 is combined with Bus31 to form the new Bus31;
- Step 7: Bus13 is combined with Bus5 to form the new Bus5;

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# Advances in Electrical and Computer Engineering

- Step 8: Bus38 is combined with Bus30 to form the new Bus30;
- Step 9: Bus6 is combined with Bus5 to form the new Bus5;
- Step 10: Bus35 is combined with Bus1 to form the new Bus1;
- Step 11: Bus22 is combined with Bus19 to form the new Bus19;
- Step 12: Bus7 is combined with Bus5 to form the new Bus5;
- Step 13: Bus16 is combined with Bus15 to form the new Bus15;
- Step 14: Bus34 is combined with Bus30 to form the new Bus30;
- Step 15: Bus28 is combined with Bus19 to form the new Bus19;
- Step 16: Bus21 is combined with Bus14 to form the new Bus14;
- Step 17: Bus30 is combined with Bus1 to form the new Bus1;
- Step 18: Bus10 is combined with Bus5 to form the new Bus5;
- Step 19: Bus24 is combined with Bus15 to form the new Bus15;
- Step 20: Bus37 is combined with Bus31 to form the new Bus31;
- Step 21: Bus29 is combined with Bus19 to form the new Bus19;
- Step 22: Bus20 is combined with Bus5 to form the new Bus5;
- Steps23: Bus25 is combined with Bus2 to form the new Bus2;
- Steps24: Bus15 is combined with Bus4 to form the new Bus4;
- Steps25: Bus31 is combined with Bus1 to form the new Bus1;
- Step 26: Bus9 is combined with Bus1 to form the new Bus1;
- Step 27: Bus26 is combined with Bus14 to form the new Bus14;
- Step 28: Bus5 is combined with Bus2 to form the new Bus2;
- Step 29: Bus17 is combined with Bus3 to form the new Bus3;
- Step 30: Bus14 is combined with Bus4 to form the new Bus4;
- Step 31: Bus36 is combined with Bus1 to form the new Bus1;
- Step 32: Bus19 is combined with Bus2 to form the new Bus2;
- Step 33: Bus27 is combined with Bus4 to form the new Bus4;
- Step 34: Bus39 is combined with Bus1 to form the new Bus1;
- Step 35: Bus4 is combined with Bus2 to form the new Bus2;
- Step 36: Bus2 is combined with Bus1 to form the new Bus1;
- Step 37: Bus3 is combined with Bus18 to form the new Bus18.
- Step 38: Bus1 is combined with Bus18 to form the new Bus18.

From the entire hierarchical cluster process analysis, Bus18 has the lowest similarity to other nodes (the farthest distance to other nodes), the fault characteristic of Bus18 is distinct. It can also be found easily out from Fig. 8 that Bus18 has remarkable difference with other buses. These results are entirely identical to the fault location set in advance, so we can also confirm exactly the fault location.

Repeated Distance Cluster Contine

Dendrogram using Average Linkage (Between Groups)

			Rescaled Distance Cluster Combine					
CASE		0	5	10	15	20	25	
Label	Num	+	+	+	+		+	
Bus22	22	-+						
Bus23	23	-+						
Bus19	19	-+						
Bus28	28	-+						
Bus29	29	-+						
Bus10	10	-+						
Bus11	11	-+						
Bus7	7	-++						
Bus8	8	-+						
Bus5	5	-+						
Bus12	12	-+						
Bus13	13	-+						
Bus6	6	-+						
Bus20	20	-+						
Bus2	2	-+ +-	+					
Bus25	25	-+	I					
Bus15	15	-+	I					
Bus16	16	-+	I					
Bus24	24	-+	1					
Bus4	4	-++	1					
Bus14	14	-+	1					
Bus21	21	-+ +-+						
Bus26	26	-+						
Bus27	27	+	+				+	
Bus1	1	-+						
Bus35	35	-+						
Bus30	30	-+	1					
Bus38	38	-+						
Bus34	34	-+						
Bus31	31	-+						
Bus32	32	-+						
Bus33	33	-+						
Bus37	37	-++						
Bus9	9	-+ +-	+					
Bus36	36	-+						
Bus39	39	+						
Bus3	3	-+		-+				
Bus17	17	-+		+			+	
Bus18	18			-+				

Figure 8. The dendrogram of hierarchical cluster analysis based on node positive sequence voltage.

## VI. CONCLUSION AND DISCUSSION

Electric power system is one of the most complex artificial systems in this world, which safe, steady, economical and reliable operation plays a very important part in guaranteeing socioeconomic development, even in safeguarding social stability. The complexity of electric power system is determined by its characteristics about constitution, configuration, operation, organization, etc. However, no matter what we adopt new analytical method or technical means, we must have a distinct recognition of electric power system itself and its complexity, and increase continuously analysis, operation and control level. The fault

## Advances in Electrical and Computer Engineering

# Volume 10, Number 1, 2010

in electric power system cannot be completely avoided. When electric power system operates from normal state to failure or abnormal state, its electric quantities (current, voltage, angles, etc.) may change significantly. In our researches, utilizing real-time measurements of PMU, based on graph theory and multivariate statistical analysis theory, we are using mainly BFS, DFS and cluster analysis, and we are seeking for the uniform laws of marked changes of electrical quantities. Then we can carry out fast and exact analysis of fault component. Finally, we can accomplish fault isolation. After comparing the identical analytical results, we can find out that all these three methods can detect faults accurately and effectively.

According to line fault and bus-bar fault (single-phase fault, phase-to-phase fault and three-phase fault) in complex electric power systems, we have carried out a great deal of simulation experiments and obtained ideal results. These researches have proven that the faults in complex electric power systems can be explored successfully by analysis and calculation based on graph theory and multivariate statistical analysis theory.

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