

# Dynamic Analysis of Permanent Magnet Synchronous Generator with Power Electronics

Yasemin ONER, Nur BEKIROGLU, Selin OZCIRA  
 Yildiz Technical University,  
 Department of Electrical Engineering, Istanbul, Turkey  
 yoner@yildiz.edu.tr, nbekir@yildiz.edu.tr, sozcira@yildiz.edu.tr

**Abstract**—Permanent magnet DC motor-generators (PMDC, PMSG) have been widely used in industrial and energy sectors recently. Power control of these systems can be achieved by controlling the output voltage.

In this study, PMDC-PMSG systems are mathematically modeled and simulated in MATLAB&Simulink software. Then the results are discussed. A low power permanent magnet synchronous generator is driven by a permanent magnet DC motor and the output voltage is controlled by a frequency cycle-converter. The output of a half-wave uncontrolled rectifier is applied to an SPWM inverter and the power is supplied to a 300V, 50Hz load. The load which is connected to an LC filter is modeled by state-space equations. LC filter is utilized in order to suppress the voltage oscillations at the inverter output.

**Index Terms**—Permanent Magnet DC Motor, Permanent Magnet Synchronous Generator, Pulse Width Modulation

## I. INTRODUCTION

Nowadays, electrical energy is usually generated by hydroelectric, thermo, and nuclear power plants as well as by renewable energy sources. Not only in energy sector, but also in land and sea transportation sectors, pure and clean electrical energy generation is rather important. Recent advancements in power electronics improved the high efficiency and high performance energy generation technologies. Due to the increasing energy costs and to the global environmental protection trends, economic and ecologic energy generation techniques involving motor-generator techniques have been an up to date technology in industry and energy sectors. The use of Permanent Magnet DC Motor&Synchronous Generator systems has recently raised interest. Particularly, permanent magnet synchronous generators are used in low speed applications. The high performance drives used in these systems are requested not to be affected by parameter deviations too much and the output voltage should have desired amplitude and frequency. In order to operate these drive systems in intended conditions, a control technique should be added on this system. In this study, a low power surface mounted permanent magnet synchronous generator with magnetic rotor structure is coupled to a motor. The mathematical model of the system is derived according to the d-q axis equations of the system and the output voltage is controlled by sinusoidal PWM technique.

## II. PMDC MOTOR - PERMANENT MAGNET SYNCHRONOUS GENERATOR SYSTEM

In the proposed system, the permanent magnet synchronous generator is coupled and driven by a permanent magnet synchronous dc motor. A frequency cyclo-converter has been used for the output voltage regulation. The motor and the generator have been mathematically modeled using d-q axis equations. The difference between permanent magnet synchronous generator and the conventional synchronous generator is the use of magnets instead of windings in the rotor. Embedded magnets or rotor surface mounted magnets can be used in rotor structure [2]. Nowadays, although there have been many different designs, generally inner embedded magnet structure is preferred in order to decrease the centrifugal effects for high speed – low power applications. In this structure, magnetic reluctance through the quadratic axis (q-axis) is higher than the magnetic reluctance through the direct axis (d-axis). In this case,  $L_d$  is lower than the  $L_q$ ; ( $L_d < L_q$ )

The PMSG with rotor surface mounted magnets used in this study can provide some more power when compared to the PMSG with inner rotor embedded magnets under the same rotor dimensions. By placing the magnets on rotor surface it can be obtained that,  $L_d = L_q$ .

In a PMSG application system, series capacitor group and bridge rectifiers with voltage regulators or converters can be used for supplying power to the loads.

## III. PERMANENT MAGNET DC MOTOR MATHEMATICAL ANALYSIS

The PMDC motor is generally preferred in applications that require compact structure and wide operating speed range. Therefore, it presents high torque characteristic in low speeds. The PMDC motor used in this study can be defined as follows:

$$U_a = R_a i_a + L_a \frac{di_a}{dt} + E \quad (1)$$

$$E = k_e \omega \quad (2)$$

$$k_e = p \psi_M \quad (3)$$

Here,  $k_e$  is the speed or voltage constant. Physical dimensions, number of winding turns and stator magnetic flux density determine  $k_e$  constant.

#### IV. PERMANENT MAGNET SYNCHRONOUS GENERATOR MATHEMATICAL ANALYSIS

The model generally used by the PMSG is the Park model. By considering only the fundamental harmonic of the flux distribution in the air-gap of the machine and by neglecting the bipolar component, the theory of the space vector gives the dynamic equations of the stator voltages as follows [3]:

$$u_{abc} = -r_s i_{abc} + \frac{d\psi_{abc}}{dt} \quad (4)$$

Three-phase axis system (abc) can be converted into two-phase (dq) axis system and the stator flux components, inductances, and the rotor magnetizing flux can be written as [4];

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} = \begin{bmatrix} L_d + L_{aa}(\theta_r) & L_{ab}(\theta_r) & L_{ca}(\theta_r) \\ L_{ab}(\theta_r) & L_d + L_{bb}(\theta_r) & L_{bc}(\theta_r) \\ L_{ca}(\theta_r) & L_{bc}(\theta_r) & L_d + L_{cc}(\theta_r) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \psi_{ma}(\theta_r) \\ \psi_{mb}(\theta_r) \\ \psi_{mc}(\theta_r) \end{bmatrix} \quad (5)$$

In Equation (5);

$L_{aa}, L_{bb}, L_{cc}$ ; are the stator self inductance of the a, b, and c phases,

$L_{ab}, L_{bc}, L_{ca}$ ; are the stator mutual inductance of the a, b, and c phases,

$\psi_{ma}, \psi_{mb}, \psi_{mc}$ ; are the rotor magnetizing flux of the a, b, and c phases.

The self and mutual inductances of the stator are dependent on sinusoidal  $\theta_r$  angle only in embedded magnet structure machines with distributed windings. Stator inductances do not depend on  $\theta_r$  in permanent magnet machines with rotor surface mounted magnets. For the undistributed windings, stator self and mutual inductances can be obtained in a similar way. However, their values would be larger than that of the machine with distributed windings [5]. The following statements can be obtained when Park transformation are applied to Equation (4) [2]. Fundamental flux equations can be calculated as:

$$\bar{\psi}_s = \psi_{ds} + j\psi_{qs} \quad (6)$$

$$\psi_{ds} = \psi_m + L_{ds} i_{ds} \quad (7)$$

$$\psi_{qs} = L_{qs} i_{qs} \quad (8)$$

$$u_{sd} = -i_{sd} R_s - L_{sd} \frac{di_{sd}}{dt} + \omega_r L_{sq} i_{sq} \quad (9)$$

$$u_{sq} = -i_{sq} r_s - L_{sq} \frac{di_{sq}}{dt} + \omega_r (L_{sd} i_{sd} + \psi_m) \quad (10)$$

$$L_{sd} = L_{ls} + \frac{3}{2} \bar{L}_m \quad (11)$$

$$L_{sq} = L_{ls} + \frac{3}{2} \bar{L}_m \quad (12)$$

Equations (6) and (7) can be rewritten as:

$$\frac{di_{sd}}{dt} = -\frac{r_s + R_L}{L_{ls} + \frac{3}{2} \bar{L}_m} i_{sd} + p i_{sq} \omega_r \quad (13)$$

$$\frac{di_{sq}}{dt} = -\frac{r_s + R_L}{L_{ls} + \frac{3}{2} \bar{L}_m} i_{sq} + \frac{\psi_m}{L_{ls} + \frac{3}{2} \bar{L}_m} p \omega_r - p i_{sd} \omega_r \quad (14)$$

Here;

$R_L$  = Phase load resistance

$\bar{L}_m$  = Average magnetic inductances

The equation of motion for a permanent magnet synchronous machine can be expressed as:

$$\frac{d\omega_r}{dt} = -\frac{3P\psi_m}{4J} i_{qs} - \frac{B_m}{J} \omega_r + \frac{1}{J} T_e \quad (15)$$

#### V. CONTROL OF FREQUENCY CONVERTER

The diode rectifier is the most simple, cheap and rugged topology used in power electronic applications. The bigger disadvantage of this diode rectifier is its disability to work in bi-directional power flow. The output dc voltage from three-phase diode bridge rectifier can be obtained from Eq. (16) where the overlap due to the internal inductance of PMSG is ignored.

$$V_{d,inv} = \frac{3\sqrt{2}V_{LL}}{\pi} \quad (16)$$

IGBT power electronic switches are triggered by comparing a triangular wave with a sinusoidal waveform with 50 Hz frequency. The frequency converter is utilized by applying the SPWM method in the inverter control system. In general, inverters are used to obtain alternating currents from the direct current. They supply power from a direct current source to an alternating current load or they can supply power to an alternating current network. Due to the control simplicity and cost effectiveness, a pulse width modulated inverter model has been considered. The Simulink block diagram of a frequency converter uncontrolled rectifier is illustrated in Fig. 1.

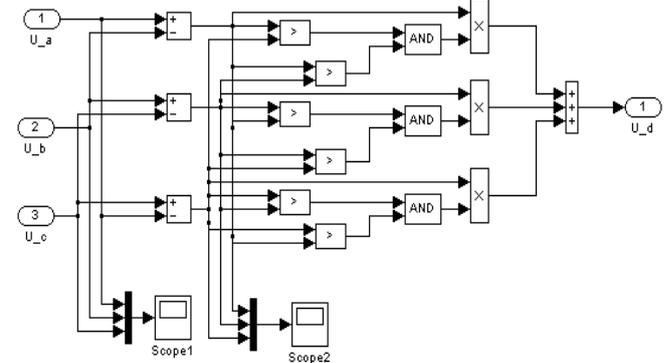


Figure 1. Simulink block diagram of an uncontrolled rectifier.

The block dc link capacitor evaluates the dc voltage at the capacitor terminals. It requires the dc-link current value, in the rectifier side ( $i_{dc-gen}$ ) and in the inverter side ( $i_{dc-load}$ ) evaluated as the quotient between the voltage supplied to load and the capacitor voltage. The difference between the dc current rectifier output side and the dc current inverter input side is the charge current of the capacitor [6]. To calculate the capacitor voltage evolution, the following equation has been applied;

$$\frac{dU_{dc}}{dt} = \frac{1}{C} (i_{dc-gen} - i_{dc-load}) \quad (17)$$

In this study improved motor-generator system is consist of permanent magnet DC motor speed and torque control

which is performed according to phase control method. Fig. 1 and Fig. 2 show speed and torque graphics respectively.

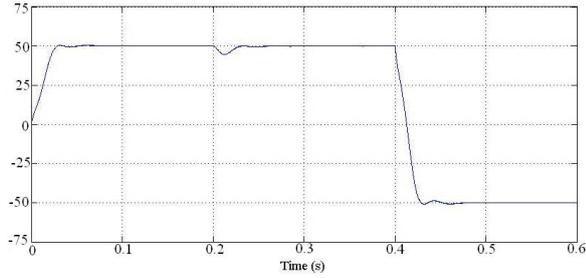


Figure 2. Permanent magnet DC motor speed.

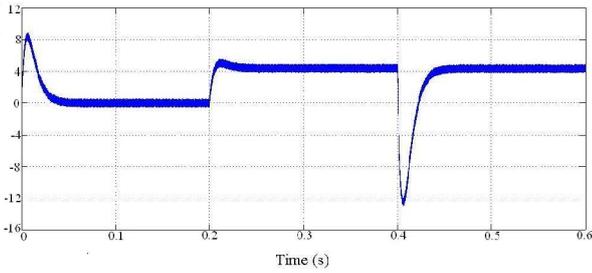


Figure 3. Permanent magnet DC motor torque.

VI. LC FILTER AND LOAD

It is desirable that the voltages and currents produced by inverter do not include harmonic components. These harmonics cause many unwanted phenomena such as noise and high dv/dt which affect the motor-generator control system as well.

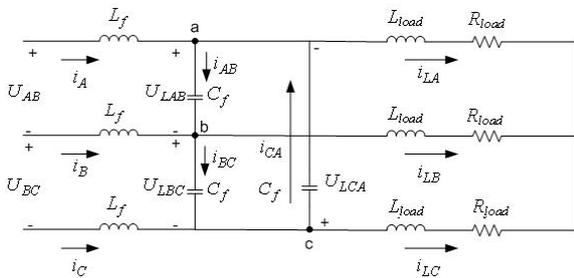


Figure 4. LC filter and load model.

$$\frac{dU_L}{dt} = \frac{1}{3C_f} I_{A0} - \frac{1}{3C_f} I_L \tag{18}$$

$$\frac{dI_{A0}}{dt} = -\frac{1}{L_f} U_L - \frac{1}{L_f} U_{A0} \tag{19}$$

$$\frac{dI_L}{dt} = -\frac{1}{L_{load}} U_L - \frac{R_{load}}{L_{load}} I_L \tag{20}$$

In equations (18), (19) and (20) the continuous steady-state equations can be written as;

$$U_L = [U_{LAB} \ U_{LBC} \ U_{LCA}]^T, I_{A0} = [I_{AB} \ I_{BC} \ I_{CA}]^T, \\ U_{A0} = [U_{AB} \ U_{BC} \ U_{CA}]^T, I_L = [I_{LAB} \ I_{LBC} \ I_{LCA}]^T$$

$$\dot{X}(t) = AX(t) + Bu(t) \tag{21}$$

$$X = \begin{bmatrix} U_L \\ I_{A0} \\ I_L \end{bmatrix}_{9 \times 1}, A = \begin{pmatrix} 0_{3 \times 3} & \frac{1}{3C_f} I_{3 \times 3} & -\frac{1}{3C_f} I_{3 \times 3} \\ -\frac{1}{L_f} I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ \frac{1}{L_{load}} I_{3 \times 3} & 0_{3 \times 3} & -\frac{R_{load}}{L_{load}} I_{3 \times 3} \end{pmatrix}_{9 \times 9}$$

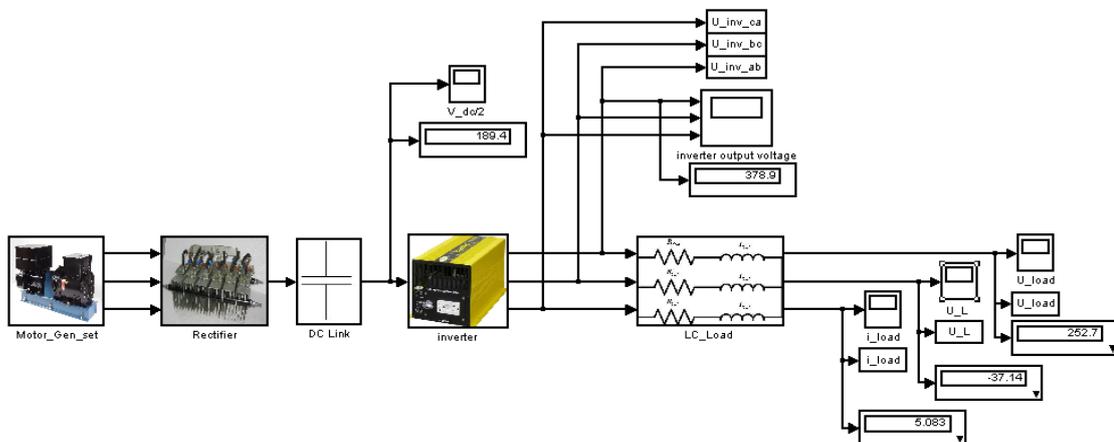
$$B = \begin{bmatrix} 0_{3 \times 3} \\ \frac{1}{L_f} I_{3 \times 3} \\ 0_{3 \times 3} \end{bmatrix}_{9 \times 3} \text{ and } u = [U_{A0}]_{3 \times 1}$$

- UL; Line voltage of load,
- IA0; Inverter output current,
- IL; Load current,
- UA0; Inverter output voltage [7].

TABLE I. MOTOR & GENERATOR PARAMETERS

Parameters	PMDC Motor values	PMSG Values
p	4	4
Ra, Rs	0.95Ω	0.3Ω
La, Lsq = Lsd	17.5mH	50mH
ΨM	0.06 Wb	0.33Wb
J	0.0063kgm <sup>2</sup>	0.0107kgm <sup>2</sup>
Bm	0.00115 Nm/rad/s	0.002044 Nm/rad/s

Figure 5. Simulink diagram of proposed Motor-Generator system output voltage control system.



VII. MODEL VERIFICATION AND SIMULATION RESULTS

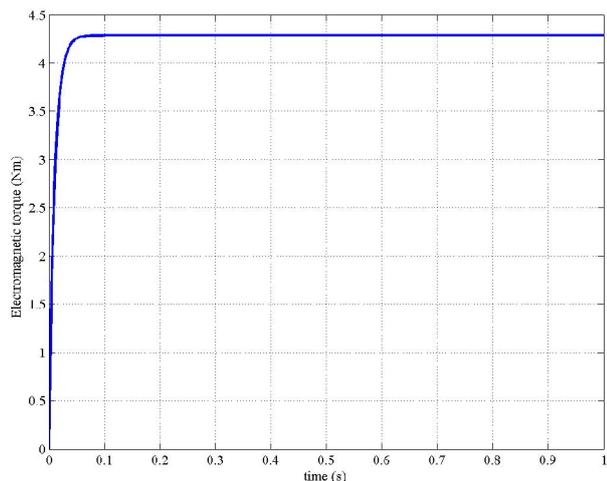


Figure 6. Permanent magnet synchronous generator torque.

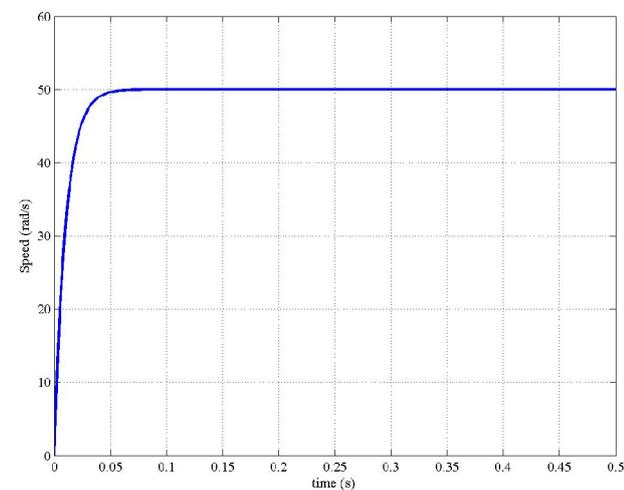


Figure 7. Permanent magnet synchronous generator speed.

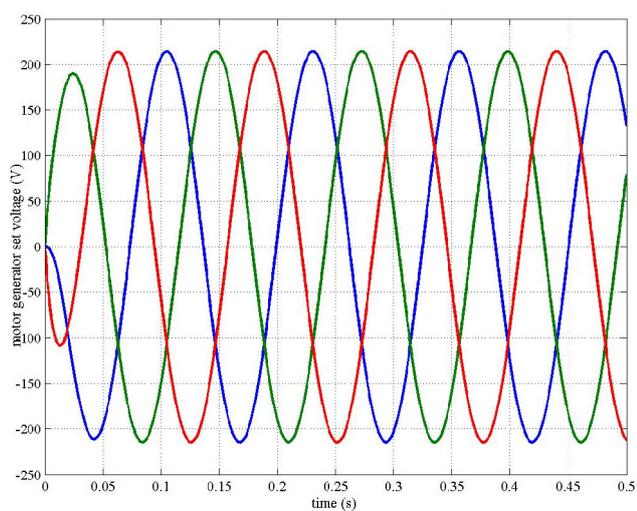


Figure 8. Motor-Generator system output voltage.

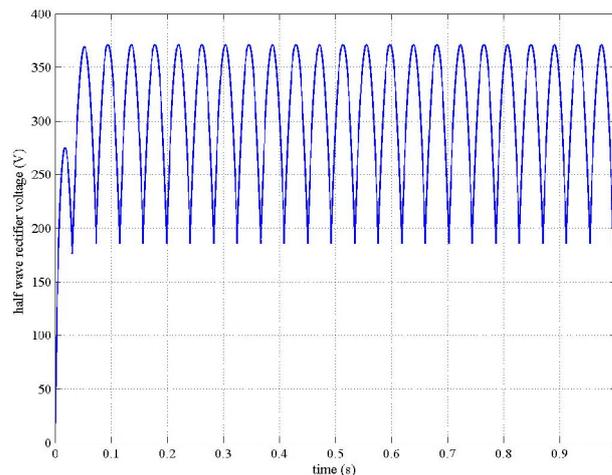


Figure 9. Half wave rectifier output voltage.

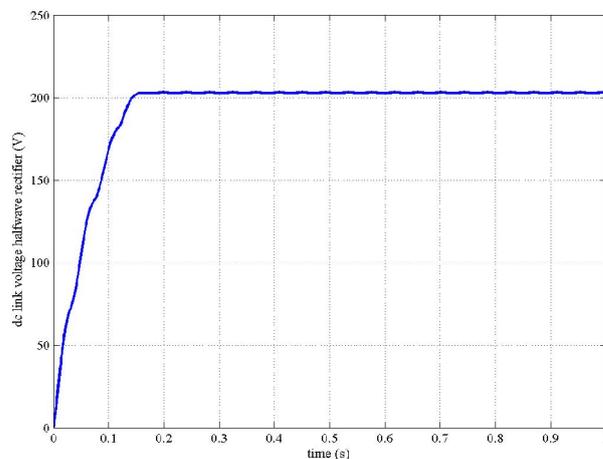


Figure 10. DC link voltage.

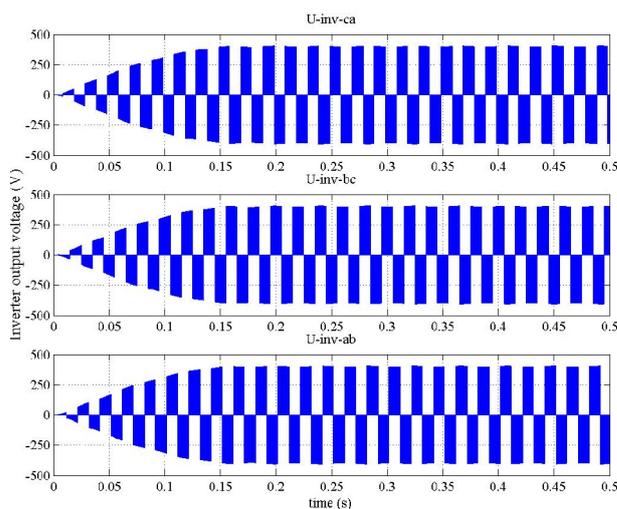


Figure 11. Inverter output voltage.

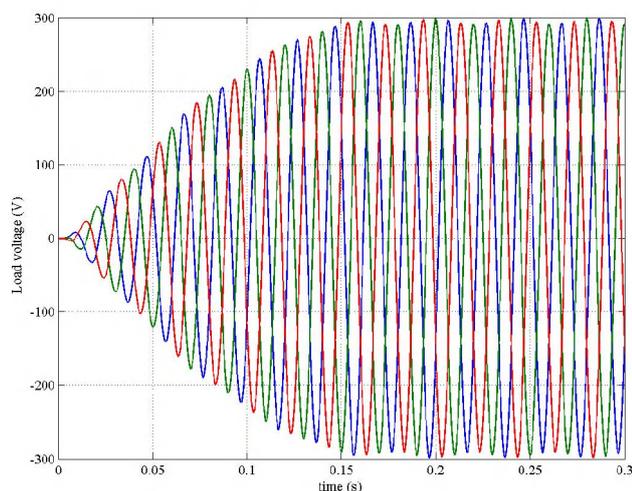


Figure 12. Load voltage.

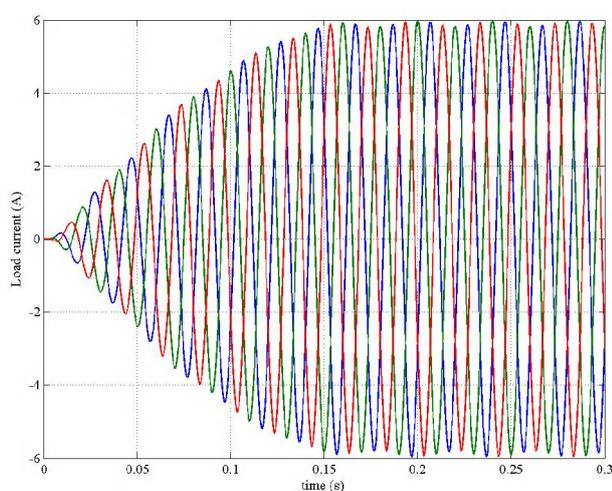


Figure 13. Load current.

### VIII. CONCLUSION

In this study, the voltage control of a permanent magnet synchronous generator has been investigated and realized for electric power generation applications. The generator is driven by a permanent magnet DC motor. The overall system and components have been mathematically modeled in MATLAB&Simulink platform and the system has been

simulated. According to the simulation results, it can be seen from Fig. 8 and Fig. 9 output voltage of motor-generator system and output voltage half wave rectifier are 220V and 380V respectively. This DC voltage is applied to the inverter input. At the inverter output, ac power is supplied to the 300V ac load. A high capacitor has been employed at the DC link in order to eliminate the voltage oscillations at the rectifier output. On the other hand, this can be compensated by suppressing voltage harmonics by using an LC filter on inverter's output. Thereby, a pure and clean sinusoidal waveform with 300V amplitude and 50Hz frequency is obtained which required for supplying the load. For the frequency converter, an SPWM inverter has been preferred for control simplicity and a half-wave uncontrolled rectifier is used with less number of elements and less expensive parts. These preferences helped keep the overall cost relatively low. In every step of the system modeling, only mathematical expressions are used hence the system is represented in a clear and meaningful manner. In this regard, it is theoretically proved that the system can be realized without the need for Simulink electrical blocks and the parameter sensitivity since only the mathematical model is used for obtaining the results.

### REFERENCES

- [1] M. Elosegui, and L. Fontan, "Analytical Design of Synchronous Permanent Magnet Motor/Generator", IEEE International Symposium on Industrial Electronics, Vigo, Spain, June 4-7, 2007.
- [2] S. Lyshevski, Electromechanical Systems, Electric Machines and Applied Mechatronics, CRC Press, 1999.
- [3] J. Brahma, L. Krichen, A. Ouali, "A Comparative Study Between Three Sensorless Control Strategies for PMSG in Wind Energy Conversion System", Applied Energy, vol 86, n. 9, September 2009, pp. 1565-1573.
- [4] I. Boldea, Variable Speed Generator, Taylor&Francis Group LLC, 2006.
- [5] C. Mun Ong, Dynamic Simulation of Electric Machinery, Prentice Hall PTR, 1998.
- [6] J.A. Sanchez, C. Veganzones, S. Martinez, F. Blazquez, N. Herrero, J.R. Wilhelmi, "Dynamic Model of Wind Energy Conversion Systems with Variable Speed Synchronous Generator and Full-Size Power Converter for Large-Scale Power System Stability Studies", Renewable Energy, vol 33, n.6, June 2008 pp.1186-1198.
- [7] J. Jung, Sine-PWM Inverter, Phd. Student Project, The Ohio State University, U.S.A, 2005.