An Approach to Synthesis of a Class of Electric Drives with Dual-Zone Speed Control

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***Abstract*—An approach to dual-zone speed control of DC motor electric drives is presented in this paper. The new point is that in the syntheses of controllers a combination between setting of closed-loop system poles (modal control) and optimal control through the quadratic quality criterion minimization is applied. Optimal modal control is achieved whereat a new type of complex criterion for selection of the functional is used. This approach allows taking into consideration the controlled object parameters' change at determination of the optimal modal controller coefficients when speed is regulated above the rated value. The research carried out as well as the results obtained can be used in the design, optimization and tuning of such types of drive systems.**

***Index Terms*—DC motor drive, dual-zone speed regulation, modal control, optimal control, optimal modal control**

I. INTRODUCTION

By technological reasons dual-zone speed control is often required in industrial automation. Regulation is carried out at constant motor torque until a basic speed is reached. After that, it is realized at constant power. The rated motor speed is most often regarded as boundary level and switching over of zones is carried out automatically.

With DC electric drives the armature voltage is regulated at constant magnetic flux in the first zone, while in the second zone the flux is reduced at constant armature voltage or back electromotive force (EMF) voltage [1].

In synthesis of the respective controllers usually two principal approaches to determination of their structure and parameters are applied.

The first one consists in compensation of the controlled object time-constants through nested control loops for the respective state vector constituents. Realization of such an approach is done with the method of subordinate control (cascade control structure). The driving systems with dual-zone speed control contain two subsystems. One is for the motor armature voltage including two control loops, namely an inner loop for the armature current and an external loop for the motor speed. The other subsystem is for the motor field including an inner loop for the field current and an external loop for the back EMF voltage [1, 10, 11]. The advantages of such systems are well known, namely: simplified tuning of the control loops, easy limitation of the regulated coordinates and possibility for unification of the control blocks. The main disadvantage of this approach is the dynamics reduction while control loops number increase.

The second approach consists in setting the root locations of the system characteristic equation [2-4, 6-9] through introduction of feedbacks for the controlled object state vector constituents. This method is used in synthesis of systems with modal control. With the systems of dual-zone speed control the major difficulty relates to the magnetic flux impact upon the modal controller coefficients in the second speed zone.

Among the various methods of the system theory based on the state space, two of them can be indicated as having largest application in the engineering practice. The first one relates to the modal control while the second method applies system optimization through quadratic quality criterion minimization. Often these methods are developed independently from each other.

This paper discusses an approach to synthesis of structurally optimized controllers [2, 7]. The aim is using an appropriate vector-matrix description of the controlled object, an optimal modal speed controller for the first zone to be synthesized, as well as an adaptive optimal modal controller of the back EMF voltage for the second zone.

II. VECTOR-MATRIX MODEL OF THE OBJECT

Operation of the considered class of DC motor drives with dual-zone speed control is described by two systems of differential equations, for the first and the second zone respectively.

A. Operation in the first speed zone

The vector-matrix model of the controlled object for the first speed zone is as follows:

$$\begin{bmatrix} \dfrac{d\omega}{dt} \\ \dfrac{dI_1}{dt} \\ \dfrac{dV_1}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{K_t}{J} & 0 \\ -\dfrac{K_e}{\tau_1 R_1} & -\dfrac{1}{\tau_1} & \dfrac{1}{\tau_1 R_1} \\ 0 & 0 & -\dfrac{1}{\tau_{c1}} \end{bmatrix} \begin{bmatrix} \omega \\ I_1 \\ V_1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{K_{c1}}{\tau_{c1}} \end{bmatrix} u_1 + \begin{bmatrix} \dfrac{K_t}{J} \\ 0 \\ 0 \end{bmatrix} I_l \quad (1)$$

where: ω is motor speed; I_1 - armature current of the motor; V_1 - armature voltage; K_t - torque coefficient; J - total inertia referred to the motor shaft; K_e - back EMF voltage coefficient; $\tau_1 = L_1/R_1$ - armature circuit time-constant; L_1 - armature inductance; R_1 - armature circuit resistance; K_{c1} - gain of the armature voltage power converter; τ_{c1} - time-constant of the armature voltage

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converter; u_1 - input control signal of the armature voltage converter; I_l - armature current which is determined by the respective load torque.

The following notations of state variables have been adopted for the first speed zone, namely: $x_1 = \omega$, $x_2 = I_1$ and $x_3 = V_1$. Measurable coordinate in this case is the motor speed ω, i. e. $y(t) = \mathbf{C}\mathbf{x}(t)$, where

$\mathbf{C} = [1 \quad 0 \quad 0]$, and $x^{\mathrm{T}} = [x_1 \quad x_2 \quad x_3]$.

The discrete state-space model can be represented as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u_1(k) + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} I_l(k). \tag{2}$$

In order to use the quadratic quality criterion in the process of synthesis, the system error of $e(k) = \omega_r(k) - \omega(k)$ should be formulated, where $\omega_r(k)$ is the reference motor speed.

It is assumed that both the reference and disturbance inputs are constant, i.e. $\omega_r(k) = \text{const}$ and $I_l(k) = \text{const}$. The following equation concerns the error and state variables, which are not outputs [4]:

$$\begin{bmatrix} x_{1e}(k+1) \\ x_{2e}(k+1) \\ x_{3e}(k+1) \\ x_{4e}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & a_{11} & -a_{12} & -a_{13} \\ 0 & -a_{21} & a_{22} & a_{23} \\ 0 & -a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_{1e}(k) \\ x_{2e}(k) \\ x_{3e}(k) \\ x_{4e}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ -b_1 \\ b_2 \\ b_3 \end{bmatrix} u_{1e}(k) \tag{3}$$

or

$\mathbf{x}_e(k+1) = \mathbf{A}_e \mathbf{x}_e(k) + \mathbf{b}_e u_e(k), \mathbf{x}_e(0) = \mathbf{x}_{e0}, k = 0, 1, 2, ...;$
$y(k) = \mathbf{C}_e \mathbf{x}_e(k)$, where

$$\left.\begin{aligned} & x_{1e}(k) = e(k-1) = \omega_r(k) - \omega(k-1); \\ & x_{2e}(k) = e(k) - e(e-1) = -[\omega(k) - \omega(k-1)]; \\ & x_{3e}(k) = I_1(k) - I_1(k-1); \\ & x_{4e}(k) = V_1(k) - V_1(k-1); \\ & u_{1e}(k) = u_1(k) - u_1(k-1); \\ & \mathbf{C}_e = [1 \quad 0 \quad 0 \quad 0]. \end{aligned}\right\} \tag{4}$$

Equation (3) has been used for the synthesis of both an optimal modal digital observer and the respective optimal modal speed controller.

The model of the DC drive for the first speed zone is realized according to equation (1) and it is shown in Fig. 1.

B. Operation in the second speed zone

Passage from the first to the second zone is carried out automatically, as the field weakening starts when the back EMF voltage reference value is reached.

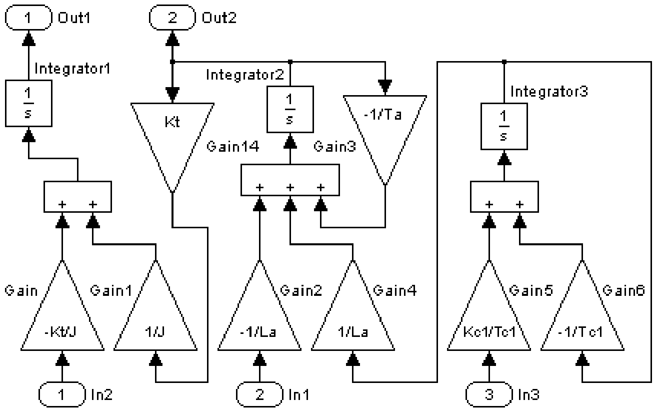

Figure 1. Model of the controlled object for the first speed zone.

The vector-matrix model of the controlled DC motor drive for the second speed zone is as follows:

$$\begin{bmatrix} \dfrac{dE}{dt} \\ \dfrac{dI_2}{dt} \\ \dfrac{dV_2}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{\tau_e} & \dfrac{K_f c\omega}{\tau_e} & 0 \\ 0 & -\dfrac{1}{\tau_2} & \dfrac{2}{\tau_2 R_2}\left(1 - \dfrac{\tau_e}{\tau_{c2}}\right) \\ 0 & 0 & -\dfrac{1}{\tau_{c2}} \end{bmatrix} \begin{bmatrix} E \\ I_2 \\ V_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{\tau_e K_{c2}}{\tau_2 R_2 \tau_{c2}} \\ \dfrac{K_{c2}}{\tau_{c2}} \end{bmatrix} u_2, \tag{5}$$

where: E is back EMF voltage; I_2 - field current; V_2 - field voltage; K_f - coefficient of the magnetic flux curve gradient; c - motor coefficient; τ_e - eddy-current time-constant; $\tau_2 = L_2/R_2$ - field circuit time-constant; L_2 - field inductance; R_2 - field circuit resistance; τ_{c2} - time-constant of the field voltage power converter; K_{c2} gain of the field power converter; u_2 - input control signal of the field voltage converter.

The following notations of state variables have been adopted for the second speed zone: $xe_1 = E$, $xe_2 = I_2$ and $xe_3 = V_2$. Measurable coordinate is the motor speed ω, i. e. $y(t) = \mathbf{C}\mathbf{x}(t)$, where

$\mathbf{C} = [1 \quad 0 \quad 0]$, and $\mathbf{xe}^{\mathrm{T}} = [xe_1 \quad xe_2 \quad xe_3]$.

The discrete state-space model of the controlled object can be represented as follows:

$$\begin{bmatrix} xe_1(k+1) \\ xe_2(k+1) \\ xe_3(k+1) \end{bmatrix} = \begin{bmatrix} ae_{11} & ae_{11} & ae_{11} \\ ae_{21} & ae_{22} & ae_{23} \\ ae_{31} & ae_{32} & ae_{33} \end{bmatrix} \begin{bmatrix} xe_1(k) \\ xe_2(k) \\ xe_3(k) \end{bmatrix} + \begin{bmatrix} be_1 \\ be_2 \\ be_3 \end{bmatrix} u_2(k). \tag{6}$$

In order to use the quadratic quality criterion in the process of synthesis, the system error of $e(k)=E_r(k)-E(k)$ should be formulated, where $E_r(k)$ is the respective reference value.

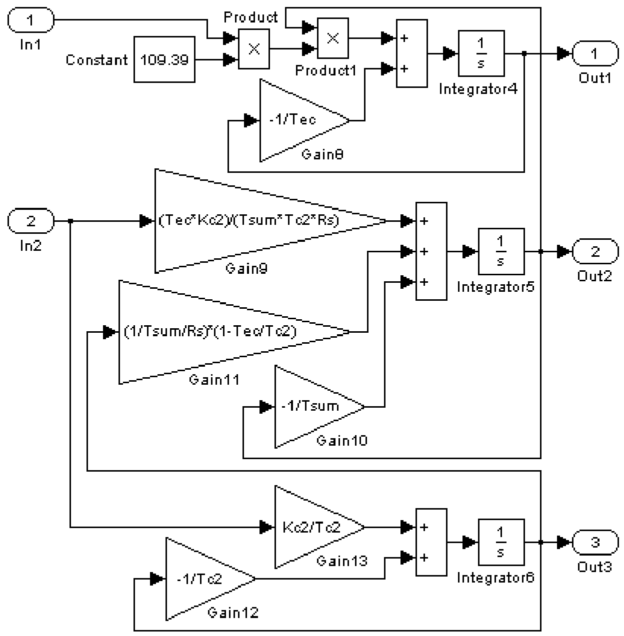

Figure 2. Model of the controlled object for the second speed zone.

It is assumed that reference input is constant, i.e. $E_r(k)=\text{const}$. The following equation concerns the error and state variables, which are not outputs [4]:

$$\begin{bmatrix} xe_1(k+1) \\ xe_2(k+1) \\ xe_3(k+1) \\ xe_4(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & ae_{11} & -ae_{12} & -ae_{13} \\ 0 & -ae_{21} & ae_{22} & ae_{23} \\ 0 & -ae_{31} & ae_{32} & ae_{33} \end{bmatrix} \begin{bmatrix} xe_1(k) \\ xe_2(k) \\ xe_3(k) \\ xe_4(k) \end{bmatrix} + \begin{bmatrix} 0 \\ -be_1 \\ be_2 \\ be_3 \end{bmatrix} u_{2e}(k) \tag{7}$$

or

$\mathbf{xe}_e(k+1)=\mathbf{A}_e\mathbf{xe}_e(k)+\mathbf{be}_e u_{2e}(k), \mathbf{xe}_e(0)=\mathbf{xe}_{e0}$, $k=0,1,2,\dots$; $y(k)=\mathbf{C}_e\mathbf{xe}_e(k)$, where

$$\left.\begin{aligned} & xe_{1e}(k)=e(k-1)=E_r(k)-E(k-1); \\ & xe_{2e}((k)=e(k)-e(k-1)=-[E(k)-E(k-1)]; \\ & xe_{3e}(k)=I_2(k)-I_2(k-1); \\ & xe_{4e}(k)=V_2(k)-V_2(k-1); \\ & u_{2e}(k)=u_2(k)-u_2(k-1); \\ & \mathbf{C}_e=[1 \quad 0 \quad 0 \quad 0]. \end{aligned}\right\} \tag{8}$$

The model of the DC motor drive for the second speed zone is developed according to equation (5) and it is shown in Fig. 2.

An optimal modal digital observer and an optimal modal speed controller for the fist zone have been synthesized, as well as an adaptive optimal modal controller of the back EMF voltage for the second zone.

The main parameters of the controlled object are presented in Table 1.

TABLE I. MAIN PARAMETERS OF THE CONTROLLED OBJECT

Parameter	Value
Rated power	3.4 kW
Rated armature voltage	220 V
Rated armature current	17.6 A
Rated speed	314 rad/s
Back EMF voltage coefficient	0.6737 Vs/rad
Torque coefficient	0.6737 Nm/A
Armature circuit resistance	1.64 Ω
Armature inductance	0.027 H
Total inertia referred to the motor shaft	0.074 kg.m^2
Armature circuit time-constant	0.0165 s
Time-constant of the armature converter	0.004 s
Field circuit resistance	365.5 Ω
Field inductance	39.2 H
Field circuit time-constant	0.11 s
Coefficient of the magnetic flux curve gradient	0.0092 Wb/A
Motor coefficient	118.19
Gain of the armature power converter	24.04
Time-constant of the field power converter	0.005 s
Gain of the field power converter	22.1

III. OBSERVER FOR THE FIRST SPEED ZONE

Synthesis of the digital observer has been carried out in accordance with an algorithm presented in [5]. The respective procedure utilizes the transpositioned additional object [7]:

$$\boldsymbol{\alpha}(k+1)=\mathbf{A}_e^{\mathrm{T}}\boldsymbol{\alpha}(k)+\mathbf{C}_e^{\mathrm{T}}(k)\beta(k) \tag{9}$$

or

$$\begin{bmatrix} \alpha_1(k+1) \\ \alpha_2(k+1) \\ \alpha_3(k+1) \\ \alpha_4(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & a_{11} & -a_{21} & -a_{31} \\ 0 & -a_{12} & a_{22} & a_{32} \\ 0 & -a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} \alpha_1(k) \\ \alpha_2(k) \\ \alpha_3(k) \\ \alpha_4(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \beta(k). \tag{10}$$

The $\mathbf{A}_e^{\mathrm{T}}$ matrix eigenvalues are determined solving the equation:

$$\det\left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & a_{11} & -a_{21} & -a_{31} \\ 0 & -a_{12} & a_{22} & a_{32} \\ 0 & -a_{13} & a_{23} & a_{33} \end{bmatrix} - \begin{bmatrix} \chi & 0 & 0 & 0 \\ 0 & \chi & 0 & 0 \\ 0 & 0 & \chi & 0 \\ 0 & 0 & 0 & \chi \end{bmatrix} \right\} = 0 \tag{11}$$

The following eigenvalues are obtained:

$$\chi_1=1,\ \chi_2=0.9960,\ \chi_3=0.9448,\ \chi_4=0.7788.$$

In this case there is one undesired root of the open-loop system ($\chi_1=1$), which must be displaced. A location for the closed-loop system root $\mu_1=0.2$ is defined, where χ_1 should be placed.

In order to define the observer $\mathbf{H}$ matrix, it is necessary to find the elements of $\mathbf{q}_1$ eigenvector, solving this system of homogenous algebraic equations:

$$(\mathbf{A_e} - \mathbf{I}\chi_i)\mathbf{q}_i = 0 \ \text{ for } i = 1 \tag{12}$$

For the elements of both eigenvector $\mathbf{q}_1$ and weight matrix $\mathbf{Q}_1$ the following is obtained:

$$\mathbf{q}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{Q}_1 = \mathbf{q}_1\mathbf{q}_1^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

These products are computed: $\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_1^{\mathrm{T}} = [1 \quad 0 \quad 0 \quad 0]$ and $\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_1^{\mathrm{T}}\mathbf{b}_e = 1$.

Weight coefficient $r_1 = 0.3125$ and the $\lambda_1 = 1.25$ coefficient are calculated. For the optimal modal feedback gain the following is obtained:

$$\boldsymbol{\gamma}_1 = [-0.8 \quad 0 \quad 0 \quad 0].$$

Since in this case there is one undesired value ($\chi_1 = 1$), the optimal modal feedback gain becomes:

$$\boldsymbol{\gamma}^* = \boldsymbol{\gamma}_1 = [-0.8 \quad 0 \quad 0 \quad 0].$$

The observer feedback vector is formulated:

$$\mathbf{H} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The observer equation is as follows [7]:

$$\hat{\mathbf{x}}_e(k+1) = \mathbf{A}_e\hat{\mathbf{x}}_e(k) + \mathbf{b}_e\mathbf{u}_e(k) + \mathbf{H}\Delta\mathbf{e}(k) = \mathbf{A}_e\hat{\mathbf{x}}_e(k) + \\ + \mathbf{b}_e\mathbf{u}_e(k) + \mathbf{H}[y(k) - \mathbf{C}\hat{\mathbf{x}}(k)]$$

or

$$\begin{bmatrix} \hat{x}_{1e}(k+1) \\ \hat{x}_{2e}(k+1) \\ \hat{x}_{3e}(k+1) \\ \hat{x}_{4e}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_{1e}(k) \\ \hat{x}_{2e}(k) \\ \hat{x}_{3e}(k) \\ \hat{x}_{4e}(k) \end{bmatrix} + \\ + \begin{bmatrix} 0 \\ -b_1 \\ b_2 \\ b_3 \end{bmatrix} u_e(k) + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} \Delta e(k) \tag{13}$$

where $\Delta e(k) = y(k) - \mathbf{C}\hat{\mathbf{x}}(k)$.

These equations produce the state variables valuation. Based on them a model of the optimal modal observer has been developed and its block diagram is shown in Fig.3.

IV. SPEED CONTROLLER

Synthesis of the optimal modal controller has been realized by an algorithm described in [4]. In this case it is carried out based on equation (3).

For the matrix $\mathbf{A}_e$ eigenvalues, the following is obtained:

$$\chi_1 = 7788, \ \chi_2 = 0.9960, \ \chi_3 = 0.9448, \ \chi_4 = 1.$$

Among these values one undesired root exists ($\chi_3 = 1$), which should be displaced.

A location for the closed-loop system root $\mu_4 = 0.8$ is defined, where χ_4 should be placed.

In order to determine the optimal modal controller matrix $\mathbf{K}$, it is necessary to find the elements of the eigenvector $\mathbf{q}_4$ solving the system of homogenous algebraic equations:

$$\left(\mathbf{A}_e^{\mathrm{T}} - \mathbf{I}\chi_{\mathrm{i}}\right)\mathbf{q}_{\mathrm{i}} = 0 \text{ for } i = 4. \tag{14}$$

The elements of eigenvector $\mathbf{q}_4$ and weight matrix $\mathbf{Q}_4$ are obtained as follows:

$$\mathbf{q}_4 = \begin{bmatrix} 0.0037 \\ 0.9887 \\ -0.1479 \\ -0.0219 \end{bmatrix},$$

$$\mathbf{Q}_4 = \begin{bmatrix} 0.0000 & 0.0036 & -0.0005 & -0.0001 \\ 0.0036 & 0.9776 & -0.1463 & -0.0217 \\ -0.0005 & -0.1463 & 0.0219 & 0.0032 \\ -0.0001 & -0.0217 & 0.0032 & 0.0005 \end{bmatrix}.$$

Next products are calculated:

$\mathbf{b}_e^{\mathrm{T}}\mathbf{Q}_4 = [-0.0005 \quad -0.1302 \quad 0.0195 \quad 0.0029]$ and $\mathbf{b}_e^{\mathrm{T}}\mathbf{Q}_4\mathbf{b}_e = 0.0173$.

For these coefficients the following values are obtained: $r_4 = 0.3469$ and $\lambda_4 = 5$.

The optimal modal feedback gain is determined:

$$\boldsymbol{\gamma}_1 = [0.0056 \quad 1.5015 \quad -0.2246 \quad -0.0333].$$

Since in this case there is one undesired value ($\chi_1 = 1$), the optimal modal feedback gain becomes:

$$\boldsymbol{\gamma}^* = \boldsymbol{\gamma}_1 = [0.0056 \quad 1.5015 \quad -0.2246 \quad -0.0333].$$

The feedback vector obtains this form:

$$\mathbf{K} = [k_1 \quad k_2 \quad k_3 \quad k_4] = \boldsymbol{\gamma}^*.$$

and control of the following type is formulated:

$$u_{1e}(k) = \mathbf{K}\mathbf{x}_e(k) = k_1x_{1e} + k_2x_{2e} + k_3x_{3e} + k_4x_{4e} \tag{15}$$

After substitution of $u_{1e}(k)$ in equation (4), for the optimal modal controller this expression is obtained:

$$u_1(k) = u_1(k-1) + k_1 x_{1e} + k_2 x_{2e} + k_3 x_{3e} + k_4 x_{4e} \qquad (16)$$

Based on equation (16), the model of an optimal modal controller has been constructed and it is shown in Fig. 4.

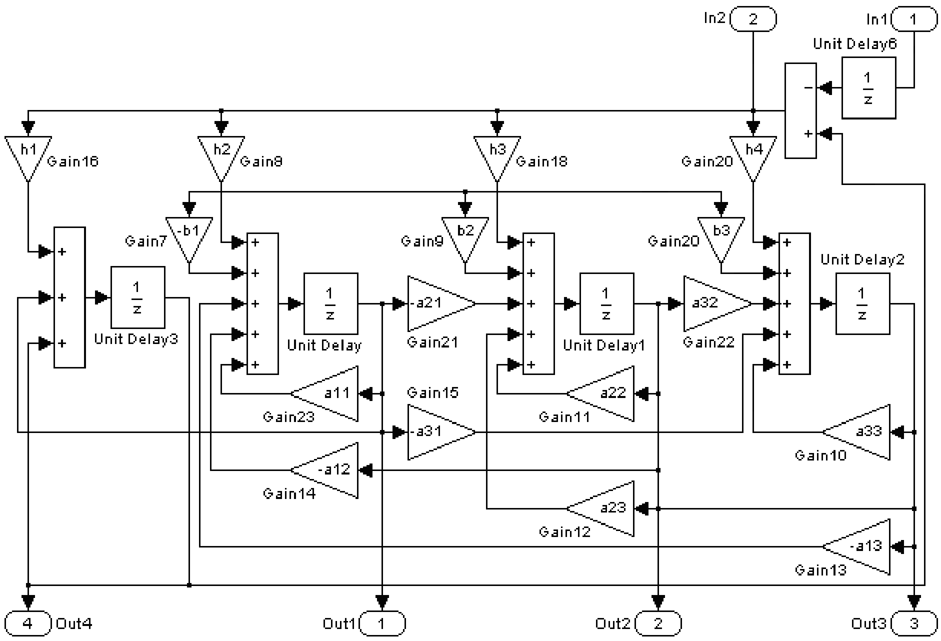

Figure 3. Model of the optimal modal observer.

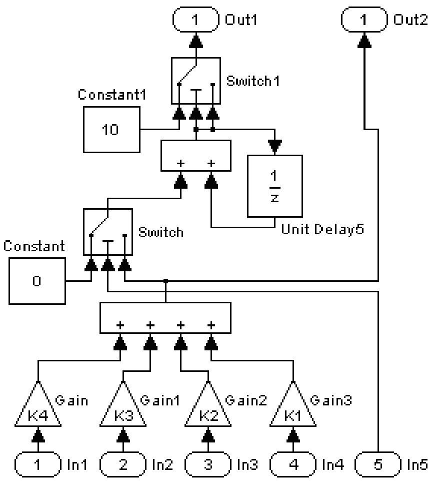

Figure 4. Model of the optimal modal speed controller.

A current limitation has been applied using the respective function as follows:

$$u_{cl}(k) = u_n + K_m \omega(k), \qquad (17)$$

where u_n is the current limitation initial code, and K_m – scale coefficient.

Therefore, the control condition in the presence of current limitation will be:

$$u_c(k) = \begin{cases} u_1(k) & \text{for} \quad u(k) \le u_{cl}(k); \\ u_{cl}(k) & \text{for} \quad u(k) > u_{cl}(k). \end{cases} \qquad (18)$$

The controlling code which should be applied to the power converter control scheme is determined by condition (18). In accordance with it an armature current limitation model is composed, shown in Fig. 5.

Practically, the optimal modal speed control is achieved through consequent realization of equations. (16), (17) and (18).

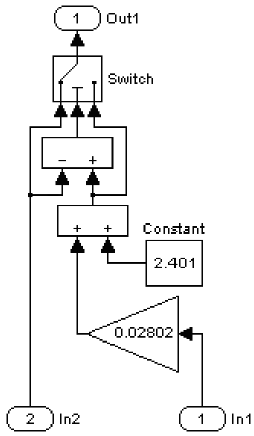

Fig. 5. Model of the current limitation.

V. BACK EMF VOLTAGE CONTROLLER

While regulating the motor speed above the basic value, back EMF voltage should be kept constant, equal to the rated value. Because the back EMF voltage cannot be measured directly, it should be obtained through calculation.

The back EMF voltage can be determined through the motor angular speed and field current after their measurement. In this case the first equation of the system (5) is used:

$$\frac{dE}{dt} = -\frac{1}{\tau_e} E + \frac{K_f c \omega}{\tau_e} I_2 . \qquad (19)$$

For small quantization periods T, equation (19) can be transformed into the next form:

$$E(k) = \frac{\tau_e}{\tau_e + T} E(k-1) + \frac{K_f c T}{\tau_e + T} \omega(k) I_2(k). \tag{20}$$

Based on equation (20) a discrete model of the back EMF voltage has been developed, shown in Fig. 6.

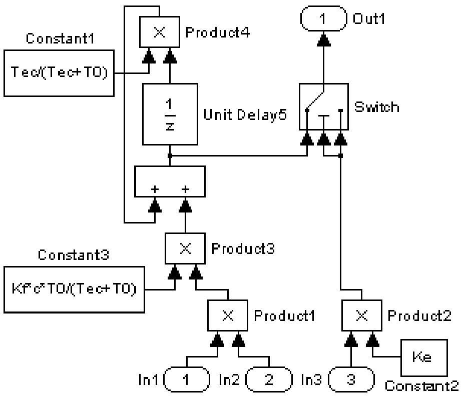

Figure 6. Model of the back EMF voltage calculator.

Synthesis for the second speed zone is realized based on equation (7). The controlled object is not stationary, because some elements of matrix $\mathbf{A}_e$ are functions of the speed ω. Therefore the optimal modal coefficients of the back EMF voltage controller will depend of the speed ω. The synthesis has been carried out by an algorithm given in [4] and the adaptive optimal modal controller is represented by the following relation:

$$\begin{aligned} u_2(k) &= u_2(k-1) + Ke_1(\omega).xe_{1e} + Ke_2(\omega).xe_{2e} + \\ &+ Ke_3(\omega).xe_{3e} + Ke_4(\omega).xe_{4e} \end{aligned} \tag{21}$$

To realize this controller some information about the object coordinates is necessary. The definition of these coordinates is based on the real measurable coordinates of the object through equations (8). Using equations (7), (8) and (20) the subsystem for determination of coordinates xe_{1e}, xe_{2e}, xe_{3e} and xe_{4e} is carried out, after measurement of ω, I_2 и V_2.

Information about the controller coefficients is also necessary. In the $\omega_{\text{rat}} < \omega \le 2\omega_{\text{rat}}$ range these coefficients are determined according to the algorithm illustrated in [4].

The optimal modal coefficients are functions of the motor speed and the respective nonlinear relationships of $Ke_1(\omega)$, $Ke_2(\omega)$, $Ke_3(\omega)$ and $Ke_4(\omega)$ can be approximated into linear ones, described by the following equations:

$$\left.\begin{aligned} Ke_1 &= -0.0000031\,\omega + 0.0079 \\ Ke_2 &= -0.00083\,\omega + 2.1204 \\ Ke_3 &= 0.000124\,\omega - 0.3171 \\ Ke_4 &= 0.000018\,\omega - 0.0468 \end{aligned}\right\} \tag{22}$$

Taking into consideration the applied current limitation, the control is realized on the bases of equations (17), (18), (21) and (22). The controller output (Fig. 7) depends on whether $\omega \le \omega_{\text{rat}}$ or $\omega > \omega_{\text{rat}}$. If $\omega \le \omega_{\text{rat}}$, $u(k) = 10$; if $\omega > \omega_{\text{rat}}$, $u(k)$ is determined by equation (17).

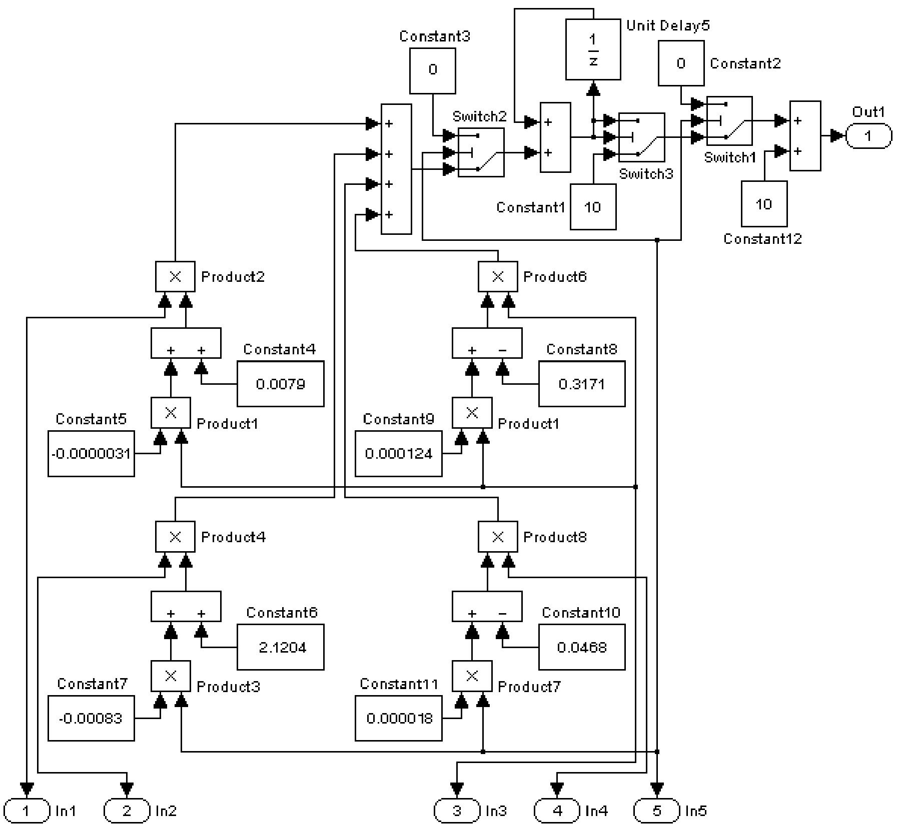

Figure 7. Model of the adaptive optimal modal controller of the back EMF voltage.

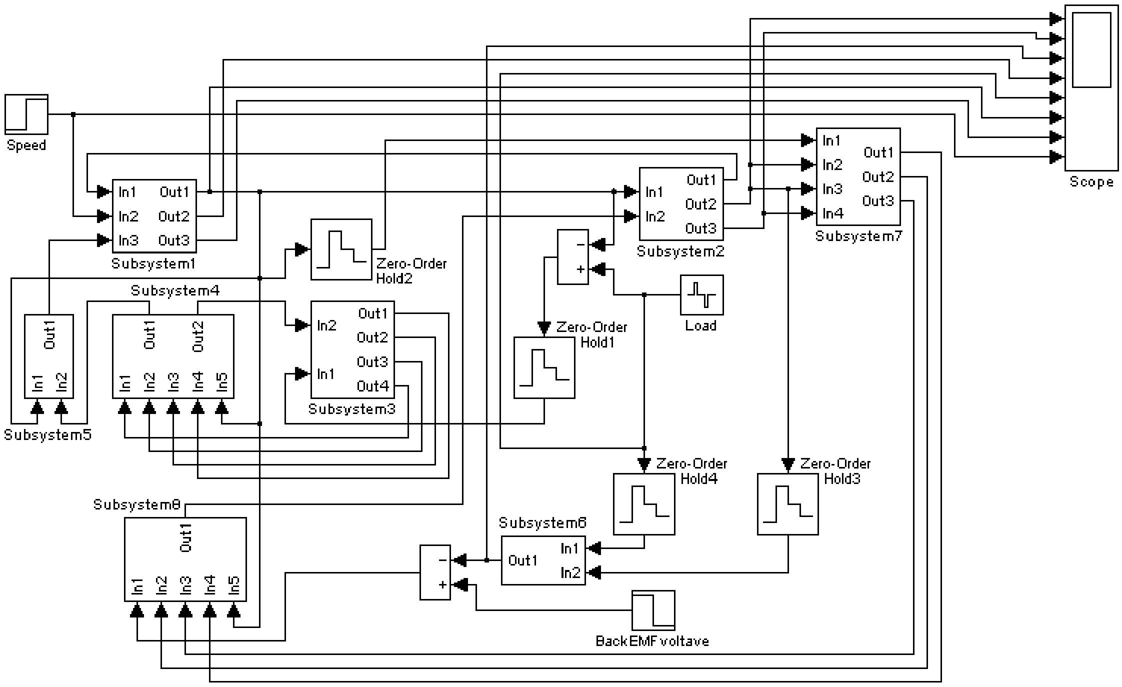

Figure 8. Model of the complete driving system with dual-zone speed control.

VI. SIMULATION AND PERFORMANCE ANALYSIS

Every working regime of the driving system has specific features. To verify the described approach functionality some computer simulation models have been developed, using the MATLAB/SIMULINK software package.

A model of complete drive system with dual-zone speed control is presented in Fig. 8, where the following notations are used: Subsystem 1 is the controlled object for the first zone; Subsystem 2 – the controlled object for the second zone; Subsystem 3 – the optimal modal observer; Subsystem 4 – the optimal modal speed controller; Subsystem 5 – the current limitation; Subsystem 6 – the back EMF voltage calculator; Subsystem 7 – the calculator of the xe_{1e}, xe_{2e}, xe_{3e} and xe_{4e} coordinates; Subsystem 8 – the adaptive optimal modal controller of the back EMF voltage. The applied quantization period in this model is $T_0 = 0.001\,\text{s}$.

Detailed investigation has been carried out for the respective dynamic and static regimes at various working conditions and loads applied to the motor shaft.

Fig. 9 shows some simulation results illustrating the performance of the driving system under consideration in the first speed zone. In this case the reference speed is $\omega_r = 160$ rad/s $< \omega_{\text{rat}}$ and the load torque applied to the motor shaft is equal to the rated value ($I_{l\text{rat}} = 17.6$ A). The real motor speed ω, the armature current I_1, the back EMF voltage E and the field current I_2 are presented. The starting armature current is limited to the maximum admissible value of $I_{1\max}$, which provides for a maximum starting torque. In this zone the current I_2 is kept constant at the rated value ($I_2 = I_{2rat}$) and the back EMF voltage is proportional to the motor speed ($E < E_{rat}$).

Time-diagrams obtained at reverse speed control with electrical braking are shown in Fig. 10. The reference speed values are 100 rad/s and -200 rad/s respectively. During the transient processes the armature current is limited to the maximum admissible value ($I_1 = \pm I_{1\max}$), which ensures good dynamics. As obvious, in this driving system the speed direction change is carried out through the armature voltage while the field current I_2 remains unchanged in both value and polarity.

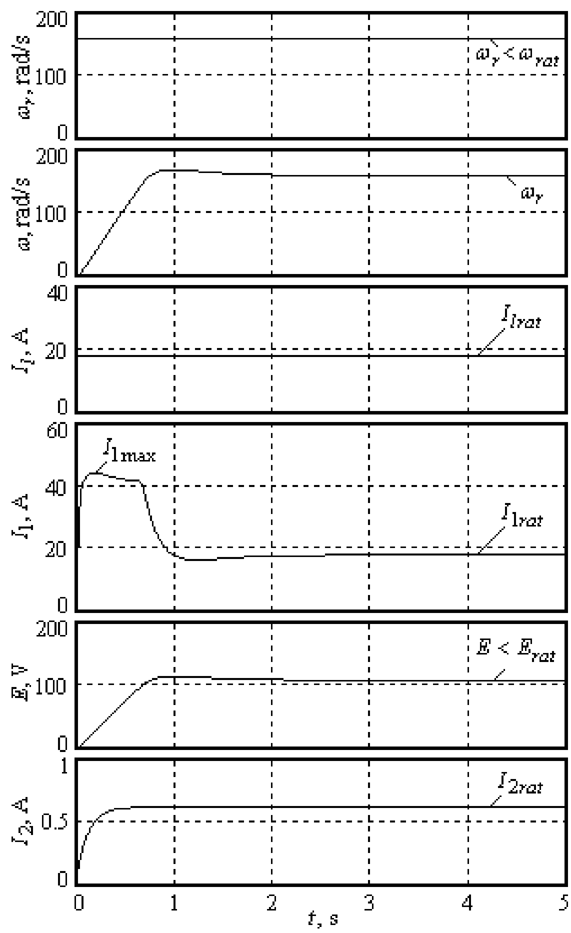

Figure 9. Time-diagrams obtained for the first speed zone.

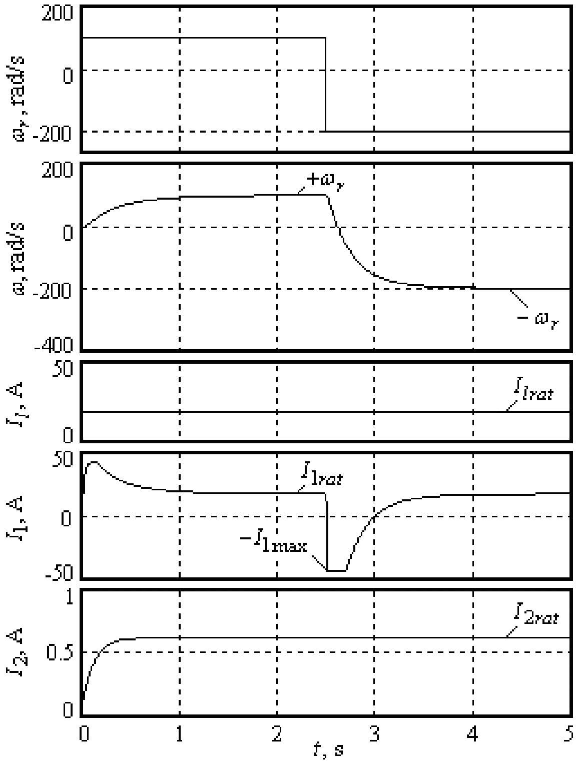

Figure 10. Time-diagrams illustrating reverse with electrical breaking.

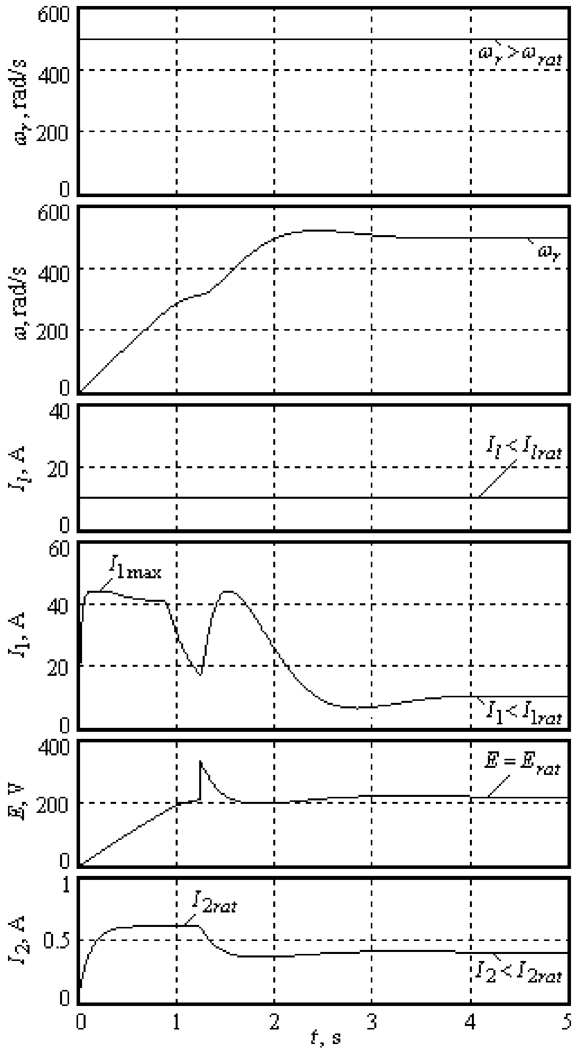

Figure 11. Time-diagrams obtained for the second speed zone.

The respective time-diagrams obtained for the second speed zone are presented in Fig. 11. In this case the reference speed is $\omega_r = 500$ rad/s $> \omega_{\mathrm{rat}}$ and the load torque applied to the motor shaft is less then the rated value ($I_l = 10\,\mathrm{A} < I_{l\mathrm{rat}}$). Passage to this zone brings to field weakening ($I_2 < I_{2rat}$) and the back EMF voltage is maintained at the rated value ($E = E_{rat}$).

VII. CONCLUSION

The proposed approach to synthesis of the considered electric drive systems allows combination between setting of the closed-loop system poles and optimal control through quadratic quality criterion minimization. Optimal modal control is carried out using a complex criterion for the functional choice. Such an approach makes it possible to take into account the changes of the controlled object parameters in the second speed zone and accomplishment of adaptive optimal modal control.

The computer simulation models developed offer relevant basis for investigation of the driving systems under consideration. Detailed studies carried out for the respective transient and steady state regimes of operation show that the presented algorithms provide for good performance.

The research reported here as well as the results obtained can be used in the design, optimization and tuning of such types of driving systems.

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