# Direct Torque Control of Induction Motor with Direct Calculation of Voltage Vector

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*Abstract*—Direct torque control is one of modern methods of A.C. machines control. The direct torque control methods with direct calculation of the vector voltage and vector pulse width modulator were developed and experimentally tested at the Department of Electronics. The paper describes the theoretical assumptions of developed control methods and differences from classical direct torque methods. In the paper, important quantities are shown which were measured on a real induction motor drive with digital signal processor control system.

*Index Terms*—Direct torque control, induction motor, vector pulse-width modulation, digital signal processor.

#### I. INTRODUCTION

New conception of electrical drives was influenced by the development of semiconductor components which enabled the development of modern frequency converters as the practical realization of modern control methods of A.C. drives including the vector control in the field coordinates of motor and direct torque control methods [1], [2], [8].

To give an induction motor high dynamic performance as two control principles may be applied: the vector control method or the direct torque control method (DTC). The first one was developed in the sixties. The direction of the rotating rotor flux vector is used as a reference axis for dividing the transformed stator current into a flux and torque producing component.

The second method was proposed in 1984 by M. Depenbrock and in 1986 by I. Takahashi and T. Noguchi. In recent years, several papers have been published about DTC. Also several modifications such as space vector modulated DTC that has constant switching frequency, has been presented.

Direct torque control methods of AC machines are characterized by their simplicity with regard to vector control. This simplicity allows for easy implementation on a control microcomputer. The high robustness and the ability of quick changes of motor torque, thanks to which it is possible to obtain very good dynamic qualities, are among the advantages of these methods. Main quantities (motor torque and magnetic flux) are calculated from measured values with the help of a mathematical model of the induction motor.

## II. DIRECT TORQUE CONTROL METHODS

All methods of direct torque control use a similar basic principle: a rotating magnetic field in the stator is created on the basis of a calculating algorithm with the help of voltage vectors  $\underline{u}_{1}$  to  $\underline{u}_{6}$  (Fig. 2), where the rotation speed of the magnetic field, and subsequently the amount of motor torque, can be controlled by the switching of zero vectors or

the switching of active vectors working in the opposite direction of the magnetic field [1].

Stator flux  $\Psi_1$  and internal motor torque *T* are calculated on the basis of the measured voltage of the DC link, switching combination and phase currents by following equations:

$$\psi_{1\alpha} = \int (u_{1\alpha} - R_s \, i_{1\alpha}) dt \tag{1}$$

$$\psi_{1\beta} = \int \left( u_{1\beta} - R_s \, i_{1\beta} \right) dt \tag{2}$$

$$\left|\psi_{1}^{S}\right| = \sqrt{\psi_{1\alpha}^{2} + \psi_{1\beta}^{2}} \tag{3}$$

$$T = \frac{3}{2} p_p \left( \psi_{1\alpha} \ i_{1\beta} - \psi_{1\beta} \ i_{1\alpha} \right) \tag{4}$$

In Takahashi Method (T\_DTC) of direct torque control, the magnetic flux of the stator is controlled so that the final point of the vector of stator magnetic flux moves within a circular ring, whereby its trajectory in a simplified case approaches a circle.

The control structure is shown in Fig. 1. Description of the blocks is as follows: FC - Frequency Converter, IM – Induction Motor, HW - Hardware, SW - Software, BVR -Block of Voltage Reconstruction, BSP - Block of Switching Pulses, BSFC - Block of Stator Flux Calculation, BT3/2 -Block of Transformation 3/2, BTC - Block of Torque Calculation, BSPC - Block of Switching Pulses Calculation, BVA - Block of Vector Analyzer, BSD - Block of Sector Determination.



Fig. 1. Control structure of AC drive with Takahashi method

For the control of motor excitation, the module of the space vector of magnetic flux is determined according to the relationships (1), (2), (3) and consequently is compared with

the desired value. The regulator is two-level with hysteresis. Then it must be determined in which sector the vector of magnetic flux is located. The torque control is based on a comparison of the calculated value of internal motor torque T according to the relationship (4) and the desired torque value  $T_{ref}$ . The controller is three-level with hysteresis [2].



Fig. 2. Principle of selection of voltage vector in the first sector of the plane  $\alpha$ ,  $\beta$  according to Takahashi Method

The control structure of modified method (M\_DTC) is the same as in previous Takahashi Method. The difference of this method is in the shifting of the sectors and in the content of the switching table, on the basis of which the selection of the voltage vector is made.



Fig. 3. Principle of selection of voltage vector in the first sector of the plane  $\alpha$ ,  $\beta$  according to Modified Method

The first sector is shifted from the original  $-30^{\circ}$  to  $30^{\circ}$  to the new  $0^{\circ}$  to  $60^{\circ}$  (see Fig.2 and Fig.3). In determining the switching table, we attempt to avoid vectors with which it is unclear whether they will cause in a particular sector an increase or decrease of torque (flux). The following table shows the influence of the switching of individual vectors in the first sector for Takahashi and the Modified Method.

The abbreviations *TI* and *TD* denote respectively the increase and decrease of torque and *FI* and *FD* denote the increase and decrease of flux.

From Table 1 we can see that with Takahashi Method, vectors  $\underline{u}_1$  and  $\underline{u}_4$  are not used, because it is not clear whether they will cause an increase or decrease of torque in a given sector. Their influence is dependent on the position of the vector of magnetic flux. In selecting the vector according to the Modified Method, vectors  $\underline{u}_3$  and  $\underline{u}_6$  are not used. In switching these vectors it is not clear whether there

will be an increase or decrease of flux. This is considered to be the main advantage of the Modified Method compared to the Takahashi Method [3].

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	Takahashi Method	Modified Method		
	$-30^{\circ}_{\cdot} \rightarrow 30^{\circ}$	$0^{\circ} \rightarrow 60^{\circ}$		
<u><b>u</b></u> <sub>1</sub>	Torque ambiguity	TD , FI		
<u><b>u</b></u> <sub>2</sub>	TI , FI	TI , FI		
<u>u</u> 3	TI , FD	Flux ambiguity		
<u><b>u</b></u> <sub>4</sub>	Torque ambiguity	TI , FD		
<u>u</u> 5	TD , FD	TD , FD		
<u><b>u</b></u> <sub>6</sub>	TD , FI	Flux ambiguity		

TABLE I. INFLUENCE OF SWITCHING OF INDIVIDUAL VECTORS IN THE FIRST SECTOR OF PLANE  $\alpha$   $\beta$ 

The switching table of the Takahashi and Modified Methods can be determined on the basis of Fig. 2, Fig. 3 and Table 1. Each of these tables contains 36 elements.

## III. METHOD WITH DIRECT CALCULATION OF VOLTAGE VECTOR

At the our research workplace, DTC method with a direct calculation of voltage vector (DVC\_DTC) was developed, which provides directly the voltage vector to be switching for each position of the stator flux without the need to divide the plane  $\alpha$ ,  $\beta$  into sectors and to create switching table as in previous methods (see Fig. 4).



Fig. 4. Trajectory of stator magnetic flux vector according to the method with direct calculation of the vector voltage

In the figure 4 we can see components  $g_1$  and  $g_2$ . The component  $g_1$  is coaxial with the stator magnetic flux vector, while the  $g_2$  is always perpendicular to the flux vector. Vector sum of these two components determines the resultant vector g. If the voltage vector will be chosen, whose direction is identical with the direction of the vector g, then it is apparent that the size of the component  $g_1$  determines motor excitation and size of the component  $g_2$  determines the degree of speed of the stator flux vector and thus size of the torque.

$$\Delta \Psi_1 = \Psi_{lref} - |\Psi_1^{S}| \tag{5}$$

- $\Delta T = T_{ref} T \tag{6}$
- $g_I = k_1 \Delta \Psi_1 \tag{7}$   $g_2 = k_2 \Delta T \tag{8}$

The vector g specifies the direction of the voltage vector which should be switched. The constants  $k_1$  and  $k_2$  are the weighting coefficients. Since the voltage inverter has only eight voltage vectors, two of which are zero, it is necessary to select the vector that is the closest to desired direction of the vector g, which can easily make that will determine in which sector the vector g is, and according to the sector the voltage vector will be assigned, which also lies in the same sector (e.g. sector III. => vector  $u_2$ , see Fig. 4).

In the case that no reversal proceeds and vector  $g_2$  is turned against the direction of rotation of the stator magnetic flux vector, is given priority to a zero voltage vector, thus torque ripple and distortions of the stator voltage are reduced.

The control structure is shown in Fig. 5. Description of each block is the same as in Figure 1.



Fig. 5. Control structure of AC drive - DTC method with direct calculation of voltage vector

The following figure 6 shows the internal structure of the block BSPC which is used for calculation of switching pulses (see Fig. 5).



Fig. 6. Internal structure of the block BSPC - Block of Switching Pulses Calculation

The basis of the block BSPC is the vector sum, which results in components dx, dy of the vector g expressed in the two-axis coordinate system  $[\alpha, \beta]$ . We can write for the vector sum with respect to maintaining the direction:

$$dx = \Psi_{l\alpha} \cdot |\mathbf{g}_1| - \Psi_{l\beta} \cdot |\mathbf{g}_2| \tag{9}$$

$$dy = \Psi_{l\beta} \cdot |\mathbf{g}_1| + \Psi_{l\alpha} \cdot |\mathbf{g}_2| \tag{10}$$

The direction and magnitude of the vector g corresponds to the desired voltage vector. Since the voltage inverter is able to provide only 6 active vectors is the next part of the program calculated the vector, which is ideal vector g comes closest. This rough selection of voltage vector causes during the control period greater flux and torque deviation. These deviations are compensated by another voltage vector in the next control period.

## IV. IMPLEMENTATION OF VPWM MODULATOR

All of the above DTC methods are characterized by variable switching frequency of converters. Although the timer of the processor set to a constant period for calculating of the control algorithms, the converter switching frequency is variable. The cause of this phenomenon is that the result of each calculation period is the voltage vector which should be switched. Because of the three-phase inverter allows to create only eight voltage vectors, so it often happens, that the same switching vector remains in several consecutive periods. A filtration of the volatile switching frequency is difficult, and therefore the level of interference is large.

The constant switching frequency can be achieved by the implementation of vector pulse-width modulator (VPWM) into the control structure. For this adaptation, the control structure with direct calculation of voltage vector appears to be the most appropriate [1], [9].

The basic idea of VPWM implementation is following. Using VPWM modulator, which accurately approximates the vector g, it gets to an optimal overcompensation of the stator flux and torque deviations, and therefore also the minimum ripple of these variables. Basic block diagram is shown in the figure 7. The symbols S1 - S6 represent switching devices of the frequency converter.



Fig. 7. Principle block diagram of VPWM modulator implementation

To realize the method with VPWM modulator, we must make the following modifications:

1) The weighting coefficients  $k_1$  and  $k_2$  are replaced by PI controllers. The output voltage varies depending on the mechanical speed and desired motor torque. It is necessary to change the size of the vector g. Using PI controller allows a precise control of the output voltage of the frequency converter, because the integration component of the PI controller is continuously adapted to the desired output voltage and proportional component of the PI controller allows dynamic response to changes in variables.

2) Components of stator flux vector  $\Psi_{I\alpha}$ ,  $\Psi_{I\beta}$ , which enter to vector sum, are limited so that the module is constant. This reduction is described by following equations:

$$\psi_{1\alpha}^{\prime} = \frac{\psi_{1ref}}{\left|\psi_{1}^{S}\right|} \psi_{1\alpha} \tag{11}$$

$$\psi_{1\beta}^{\prime} = \frac{\psi_{1ref}}{\left|\underline{\psi}_{1}^{S}\right|} \psi_{1\beta} \tag{12}$$

where  $\Psi_{Iref}$  is reference value of stator flux and  $|\Psi_I^{S}|$  is calculated value of stator flux vector (see Eq.3). The influence of the size desired stator flux on the size of the resulting vector  $\boldsymbol{g}$  is removed by this limitation. Information about an angle of stator flux vector is preserved.

3) The components dx and dy are adjusted so that their values correspond to the value of output voltage. For the

calculation of these components, disparate values (stator flux deviation, torque deviation) are used. The final components of the vector g are therefore dimensionless numbers, which must be converted to values of corresponding voltages. For proper transfer of components, it is not possible to use any empirically derived relationship, and it is therefore necessary to experimentally determine the conversion. The value of the stator flux is calculated using the components of vector g, it is necessary to ensure that the size of this vector does not exceed the value which the VPWM modulator is able to realize. The limitation is done only when the size of the vector g exceeds a certain maximum value. The principle limitation is the same as above:

$$dx' = \frac{g_{\max}}{|g|} dx \tag{13}$$

$$dy' = \frac{g_{\max}}{|g|} dy \tag{14}$$

$$g_{\max} = \frac{U_d}{\sqrt{3}} \tag{15}$$

where  $U_d$  is voltage in DC link of the frequency converter. By these relationships, the maximum value of the basic harmonic of the output voltage at modulation ratio equal to one is calculated. This restriction prevents that vector g does not exceed the circle inscribed in a hexagon (see Fig.8).



Fig. 8. Limitation of the components dx and dy

The resulting block structure with the introduction of the adjustments is shown in figure 9.

## V. EXPERIMENTAL RESULTS

For the technical verification of the control algorithms, an electrical drive with induction motor was realized which consists of the induction motor 2.7 kW, load mechanism, frequency converter with DC link and control system with modern digital signal processor. The digital signal processor TMS320F2812 is used in the control system [6], [7].

The meaning of quantities which are shown in individual figures 10 to 20 is as follows:

T <sub>ref</sub>	 reference torque
T	 real calculated torque
i <sub>la</sub>	 phase stator current
$u_{1a}$	 phase stator voltage
$\Psi_{lref}$	 reference stator magnetic flux
$\Psi_{1\alpha}$	 $\alpha$ -component of stator flux vector
$\Psi_{1\beta}$	 $\beta$ -component of stator flux vector

In figures 10, 12, 14, 16, the courses of quantities, which characterize the properties of different direct torque control method, at the beginning of the DTC methods are shown, where at first the excitation of the motor is caused by the pulsing switching of vector  $\underline{u}_2$  to the desired flux and subsequently the control algorithm of the DTC method is started.



Fig. 9. Control structure of AC drive - DTC method with VPWM

In figures 11, 13, 15, 17, the courses of quantities at the torque command change from +6 Nm to -6 Nm are shown. In all cases, the voltage of the DC link was 200 V.



Fig. 10. Courses of quantities at run-up Takahashi Method ( $T_{ref} = 19 Nm$ ,  $\Psi_{Iref} = 0.87 Wb$ ) (ch1:  $i_{Ia} = f(t)$ , 15 A/scale; ch2:  $\Psi_{1a} = f(t)$ , 1Wb/scale; ch3:  $u_{Ia} = f(t)$ , 200 V/scale; ch4: T = f(t), 10 Nm/scale)



Fig. 11. Courses of quantities at the torque command change Takahashi Method ( $T_{ref} = 6/-6 Nm$ ,  $\Psi_{lref} = 0.7 Wb$ ) (ch1:  $i_{la} = f(t)$ , 9 A/d; ch2:  $\Psi_{1a} = f(t)$ , 1Wb/d; ch3:  $u_{la} = f(t)$ , 200 V/d; ch4: T = f(t), 10 Nm/d)



**Fig. 12.** Courses of quantities at run-up Modified DTC Method ( $T_{ref} = 19 Nm$ ,  $\Psi_{Iref} = 0.87 Wb$ ) (**ch1**:  $i_{Ia} = f(t)$ , 15 A/scale; **ch2**:  $\Psi_{1a} = f(t)$ , 1Wb/scale; **ch3**:  $u_{Ia} = f(t)$ , 200 V/scale; **ch4**: T = f(t), 10 Nm/scale)



Fig. 13. Courses of quantities at the torque command change Modified DTC Method ( $T_{ref} = 6/-6 Nm$ ,  $\Psi_{Iref} = 0.7 Wb$ ) (ch1:  $i_{Ia} = f(t)$ , 9 A/d; ch2:  $\Psi_{1a} = f(t)$ , 1Wb/d; ch3:  $u_{Ia} = f(t)$ , 200 V/d; ch4: T = f(t), 10 Nm/d)



**Fig. 14.** Courses of quantities at run-up DVC-DTC Method ( $T_{ref} = 19 Nm$ ,  $\Psi_{lref} = 0.87 Wb$ ) (**ch1**:  $i_{Ia} = f(t)$ , 15 A/scale; **ch2**:  $\Psi_{1a} = f(t)$ , 1Wb/scale; **ch3**:  $u_{Ia} = f(t)$ , 200 V/scale; **ch4**: T = f(t), 10 Nm/scale)

In figures 18, 19, 20, the stator flux trajectories of different direct torque control methods are shown.



Fig. 15. Courses of quantities at the torque command change DVC-DTC Method  $(T_{ref} = 6/-6 Nm, \Psi_{Iref} = 0.7 Wb)$ (ch1:  $i_{Ia} = f(t), 9 A/d;$  ch2:  $\Psi_{Ia} = f(t), 1Wb/d;$  ch3:  $u_{Ia} = f(t), 200 V/d;$  ch4: T = f(t), 10 Nm/d)



**Fig. 16.** Courses of quantities at run-up VPWM-DTC Method ( $T_{ref} = 19 Nm$ ,  $\Psi_{lref} = 0.87 Wb$ ) (**ch1**:  $i_{la} = f(t)$ , 15 A/scale; **ch2**:  $\Psi_{1a} = f(t)$ , 1Wb/scale; **ch3**:  $u_{la} = f(t)$ , 200 V/scale; **ch4**: T = f(t), 10 Nm/scale)



**Fig. 17.** Courses of quantities at the torque command change VPWM-DTC Method  $(T_{ref} = 6/-6 Nm, \Psi_{Iref} = 0.7 Wb)$ (**ch1**:  $i_{Ia} = f(t)$ , 9 A/d; **ch2**:  $\Psi_{1a} = f(t)$ , 1Wb/d; **ch3**:  $u_{Ia} = f(t)$ , 200 V/d; **ch4**: T = f(t), 10 Nm/d)

The following behaviors can be observed - energizing the motor and then the first revolution of the stator flux vector.



Fig. 18. Trajectory of stator flux - Takahashi Method (**ch**:  $\Psi_{1\beta} = f(\Psi_{1\alpha}), 0, 2 \text{ Wb/d}$ )



Fig. 19. Trajectory of stator flux - DVC-DTC Method (**ch**:  $\Psi_{1\beta} = f(\Psi_{1\alpha}), 0, 2 \text{ Wb/d}$ )



Fig. 20. Trajectory of stator flux - VPWM-DTC Method (**ch**:  $\Psi_{1\beta} = f(\Psi_{1\alpha}), 0, 2 \text{ Wb/d}$ )

We can express following conclusions after comparing the three methods of direct torque control.

The main advantages of DTC methods include the ability to rapid torque changes, which is confirmed in the practical measurements. A torque ripple is a consequence of the final time calculation of the control structure. In each control cycle is calculated instantaneous value of the torque and

stator magnetic flux. On the basis of these values is calculated voltage vector, which should ensure a reduction of torque and stator flux variations. This means that the torque increases or decreases over a time period depending on the sign of the previous torque deviation without the possibility of further regulation. In this case, the control creates torque deviations from the desired torque value, which are practically expressed as torque ripple.

The following factors also affect torque ripple: 1) Used method for torque reduction - it is preferable to reduce the torque with zero switching vector than the rotation of the magnetic flux vector opposite the direction in which the rotor of the induction motor rotates; 2) DC-link voltage value - increasing voltage value increases torque ripple; 3) Stator magnetic flux value - decreasing flux value reduces torque ripple; 4) Used DTC method - DTC method with VPWM achieves the lowest value of torque ripple. This estimated minimum value confirms advantages of this method based on accurate approximation of the vector g.

### VI. CONCLUSION

Properties of different direct torque control methods for induction motors are described in the paper. The direct torque control methods with direct calculation of the vector voltage and vector pulse width modulator were developed and experimentally tested at the Department of Electronics. These methods are original contributions to the direct torque control of induction motors. Experimental results confirmed the excellent properties of both methods in steady state and transient states.

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