On ECG Compressed Sensing using Specific Overcomplete Dictionaries

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***Abstract*—The paper presents a number of results regarding the construction of specific overcomplete dictionaries for ECG compressed sensing (CS). The dictionaries were built using normal and patological cardiac patterns extracted from 24 recordings of the MIT-BIH Arrhythmia Database. It has been shown that the compression results obtained using the CS concept based on specific dictionaries are better that those using the wavelet overcomplete dictionaries. Starting from the concept of sparse signal with respect to a given overcomplete dictionary the paper present several results regarding the possibility of simple pattern classification as well.**

***Index Terms*—Compressed sensing, Biomedical signal processing, Electrocardiography, Pursuit algorithms, Signal processing algorithms**

I. INTRODUCTION

The development of uni- and multi-dimensional signals acquisition, compression and processing - an extremely vast area – constituted and still constitutes the preoccupation of a large number of researchers, the goal being performances improvement and optimization according to often contradictory constraint regarding speed, precision, robustness, consumption, portability, cost, etc. [1-5]

For clinical diagnosis it is often important to record the medical data of the patients over a longer period of time or to transmit it, when the diagnostic is done remotely. In order to detect anomalies or diseases, the doctor may need 24 hour or even longer recordings. For example, for an ECG signal, its storage or transmission can reach 26MB for a one channel ECG recording with 12 bits/sample and a sampling frequency of 400Hz or even 138MB for two channels ECG recording with 16 bits/sample and the same sampling frequency. Besides, it is known that the ECG signal is also used for other cardiac monitoring and diagnostic applications, including transmission through phone channels in ambulatory monitoring and recordings in anesthesia and intensive care units of hospitals [2], [6]

The storage and transmission of large amounts of data are time and resource consuming operations which can be optimized using data compression techniques. The three important features of a compression algorithm are the compression rate (CR), the reconstruction error and the computational complexity, the first two being interdependent. The CR is defined as the ratio between the number of bits needed to represent the original and the compressed signal [7], [8] .

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For lossy compression techniques, defining the error criterion to evaluate the distortion of the reconstructed signal with respect to the original one is of paramount importance particularly for biomedical signals like the ECG where a slight loss or change of information can lead to wrong diagnostics. In most ECG compression algorithms, the percentage root-mean-square difference (PRD) measure and its normalized version, PRDN, which does not depend on the signal mean $\overline{x}$, defined as

$$PRD\% = 100\sqrt{\frac{\sum_{n=1}^{N}(x(n)-\tilde{x}(n))^2}{\sum_{n=1}^{N}x^2(n)}}$$

$$PRDN\% = 100\sqrt{\frac{\sum_{n=1}^{N}(x(n)-\tilde{x}(n))^2}{\sum_{n=1}^{N}(x(n)-\overline{x})^2}}$$

are used. In the above formulas, $x(n)$ and $\tilde{x}(n)$ are the original and the reconstructed signals respectively, $\overline{x}$ is the original signal mean and N is the length of the window over which the PRD is calculated.

In order to evaluate the relative preservation of the diagnostic information in the reconstructed signal compared to the original one, Zigel [9] introduced a new measure (which is not always easy to use) called Weighted Diagnostic Distortion (WDD) which consists in comparing the P and T wave, and QRS complexes features of the two ECG signals. Moreover, the Quality Score (QS) representing the ratio between the CR and the PRD has been recently proposed as a measure of the compression value that takes into consideration the trade-off between CR and distortion [10].

Nevertheless the final verdict regarding the fidelity and clinical acceptability of the reconstructed signal should be validated through visual inspection by the cardiologist physician.

Last but not least, the computational complexity is directly related to practical implementation considerations and is desired to be as low as possible, especially for portable equipment.

Due to the importance of the ECG signals many specific compression algorithms for this type of signals have been proposed. According to the processing method used, Zigel [11] classifies the ECG compression methods into two categories: time-domain and transform-domain compression. Time-domain compressions are fast, easy to implement and have low distortions, but also low compression ratios; this is why transform-domain methods are preferred in the scientific world, as well as in medical applications. They basically consist in representing signals in terms of appropriate basis elements or atoms in complete or over complete dictionaries respectively.

A dictionary $D = (\phi_\gamma : \gamma \in \Gamma)$ is a collection of parameterized waveforms ϕ_γ. For N-dimensional signals $\varphi_\gamma \in R^N$ are discrete-time signals of length N called atoms. Dictionaries can be complete when they contain exactly N linearly independent atoms (a basis, possibly orthonormal) or overcomplete (they contain more than N atoms). [12]

A fundamental challenge in ECG signals compression techniques is that of increasing the CR and decreasing the distortion i.e., improving the QS. For transform domain compression this means finding appropriate dictionaries and projection vectors to achieve ECG representation with as few as possible relevant atoms - a need the suggested the idea of investigating the possibility of using the recent theory of compressed sensing (CS) [13].

This paper presents an ECG signals compression method based on specific dictionaries and techniques of sparse decomposition. Using a training set of ECG signals associated to normal and several cardiac diseases a construction method of overcomplete dictionaries is presented. Based on these dictionaries, the ECG can be accurately represented by sparse vectors with entries associated with the most significant atoms and reconstructed using appropriate linear programming algorithms.

Section II is a brief presentation of the compressed sensing principles, Section III presents aspects regarding the implementation methodology and the construction method of the specific ECG signal dictionaries and the possibility of cardiac beats classification using the parameters extracted through CS. The experimental results of the ECG signals compression using CS concepts are presented in Section IV together with the results obtained using Coiflet4 type wavelet overcomplete dictionaries and dictionaries specific to the ECG signal, for two types of projection matrixes. Section V contains an overview of the previously presented results, a comparison with other ECG compression methods and is followed by conclusions.

II. PRINCIPLES OF COMPRESSED SENSING

The concept of compressed sensing (CS) [13], [14], [15] refers to the possibility of reconstructing the samples of signals which are sparse in certain bases or dictionaries. The technique, which is attractive in the case of reduced acquisition and transmission resources compared to those for decoding, seeks to answer the question "how can we achieve an efficient compression?" using the philosophy "acquire first, ask questions later" [16], [17].

In its classical form, for the 1D case the theory of CS considers an Nx1 signal x which is sparse or compressible in some basis (i.e., complete dictionary) $\Psi = \{\Psi_i, i = 1,\dots,N\}$ i.e., it can be written as $x = \Psi\alpha$ where $\alpha = \{\alpha_i, i = 1,\dots,N\}$ are rapidly decreasing to zero so that it can be approximated well enough using only the K largest coefficients, the remaining N-K being replaced by zeros. It has been proved that such a signal can be recovered from a number of nonadaptive projections on a specified set of vectors, their number being dependent on the signal dimension and its sparsity. The CS theory gives an estimation of the number of projections needed so the signal can be recovered at a similar quality to that corresponding to the approximation obtained from $M = O(K \log(N/K))$ (M is proportional to the quantity $K \log(N/K)$) non-adaptive linear projections on a second base Φ whose elements satisfy the property that neither they can be used to sparsely represent those of the initial base nor vice versa. [15], [18]

Thus, for sparse signals, instead of acquiring N samples according to the sampling theorem, a smaller number M of nonadaptive (signal independent) projections on random vectors are taken from which the signal can be then reconstruction. The reconstruction can be perfect or approximate while the number of projections depends on the signal dimension and sparsity with respect to the adopted dictionary[2]. The direct relationship between the sparsity or, equivalently, the compressibility of a signal and the number of random projections necessary for the signal reconstruction (which can be perfect in certain conditions) is apparent.

The M×1 projection vector y can be written $y = \Phi x + n = \Phi\Psi\alpha + n = \Theta\alpha + n$ where the signal n represents the cumulated effect of the quantification and of the inherent noise of the measurement. The computation of the coefficients α for the reconstruction of the signal x is an optimization problem which relies on the compressibility of x in the base Ψ. The reconstruction is made by means of standard linear programming algorithms with quadratic constraints such as LARS, LASSO, SparseLab, l_1 Magic, (Orthogonal) Basis Pursuit, (Orthogonal) Matching Pursuit etc., for which there are fast implementations [19], [20], [21], [22].

Since CS theory is very efficient provided the signals are highly sparse in an orthogonal basis – a property which is seldom satisfied – it has been extended to the case of signals that are sparse with respect to atoms in overcomplete dictionaries, i.e., with dictionaries whose elements/atoms are not linearly independent [23]. Again the signal x can be written as $x = \Psi\alpha$ where now Ψ is an N×P matrix with P>>N. Several choices for overcomplete dictionaries are: megadictionaries obtained by merging complete dictionaries (i.e., Fourier + canonical basis, Fourier + wavelet), wavelet overcomplete dictionaries, optimized/learned overcomplete dictionaries etc.). For these cases the previously mentioned methods work as well, in particular the method of basis pursuit (BP). BP finds the best representation of a signal by minimizing the l_1-norm of α for an overcomplete dictionary. Since we would like as many components of α to be zero or as close to zero as possible one have to solve the problem [12]:

[2] Basically the possibility of reconstructing signals from few projections relies on a fundamental mathematical result represented by the Johnson–Lindenstrauss lemma [24] that states that a small set of points in a high-dimensional Euclidean space can be embedded into a space of lower dimension almost conserving the distances.

$$\text{minimize} \quad \|\alpha\|_1 \quad \text{subject to } \Psi\alpha = x$$

where the nonzero components of α correspond to the dictionary waveforms on which the signal representation is based. Using the l_1 -norm replaces the LP hard problem of minimizing the l_0 norm of α with a linear programming problem (LP) of the form:

$$\text{minimize} \quad c^T\alpha \quad \text{subject to } \Psi\alpha = x, \quad \alpha \in R^N$$

where $c^T\alpha$ is the objective function and Ψ α=x can be viewed as a collection of constraints.

To solve the above equations any LP algorithm can be used; in this paper the Interior Point Method (IPM) [12] [19] has been chosen.

III. METHOD

As already mentioned, the key issue regarding an efficient CS or compressibility for a class of signals is that of finding appropriate dictionaries and projection matrices to get high compression QS's. For many classes of signals, good (time-frequency or time-scale) dictionaries for CS have been already proposed [12], [25], [26]. Still there are classes of signals like the ECG for which the use of standard dictionaries does not ensure spectacular CS results, new dictionaries are needed. The analytic construction of dictionaries such as wavelets, curvelets etc. stems from the deep mathematical tools of Harmonic Analysis [27] [12]. However, since it is difficult and time consuming to develop complex mathematical theory each class of data, the alternative solution of dictionary learning which basically consists in building the dictionary from a set of training data is the most advantageous solution.

ECG compression using CS

We have investigated three types of dictionaries namely:

- ***The Coiflet4 overcomplete wavelet dictionary (WD*)**, a time-frequency dictionary containing $N\log_2(N)$ waveforms [12]. For ECG signals with N = 256 samples the dictionary contains 2048 Coiflet atoms [28].
- ***Overcomplete dictionaries built from resampled cardiac patterns (RCP*)**: the ECG has been segmented by taking cardiac patterns between the middles of successive RR intervals. Each segment contains the P-wave, the QRS complex and the T-wave and each cardiac segment thus obtained was resampled with 301 samples using linear interpolation such that all cardiac patterns have the same dimension, and thus being possible to create a specific dictionary [29].
- ***Overcomplete dictionaries built from resampled and R-centered cardiac patterns (RRCP):*** the cardiac patterns were resampled to 301 samples and processed as follows: the R wave was elastically shrinked/stretched with respect to the peak of the R wave until it moved in the middle of the waveform support and then the left and right sides were resampled each with 150 samples such that the peak of the R wave was now positioned on sample 151. To make this type of processing reversible, the information about the initial position of the R wave peak was retained [29].

To test the compression method with the above mentioned overcomplete dictionaries 24 ECG annotated recordings from the MIT-BIH Arrhythmia database have been used [30]. The ECG signals were initially digitized through sampling at 360 samples per second, quantized and encoded with 11 bits and then resampled as described above.

Based on the database annotations, 8 major classes have been identified, namely a class of normal cardiac beats and 7 classes of pathological beats: atrial premature beat, left bundle branch block beat, right bundle branch block beat, premature ventricular contraction, fusion of ventricular and normal beat, paced beat, fusion of paced and normal beat.

The two dictionaries (RCP and RRCP) were built using randomly selected patterns form the 24 recordings of the MIT-BIH Arrhythmia database and contain 2367 cardiac patterns, evenly distributed for all the 8 classes of cardiac beats. No pattern used for the construction of the dictionary has been employed later for testing.

The CS of ECG signals assume the existence of a projection matrix containing a number of vectors of the same dimension N as the ECG signals equal to the ration between N and the imposed CR.

For the random projection matrix used in CS two cases have been considered, namely:

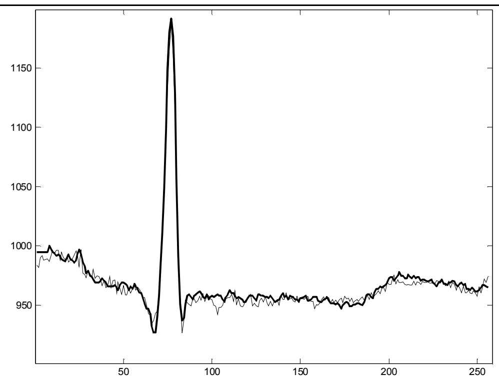
- The projection matrix contains pseudo-random values drawn from a normal distribution with mean zero and standard deviation one (These matrices were generated using the randn function in Matlab).
- The projection matrix contains only binary values, each line of the matrix (each projection vector) containing 50 ones and 50 zeros, and the positions of the ones values being randomly generated with a uniform distribution.

For the ECG signals reconstruction, the Basis Pursuit (BP) method has been used. As previously mentioned this method minimizes the l_1 norm of the α coefficients and has been shown to be optimal method from the point of view of the reconstruction errors.

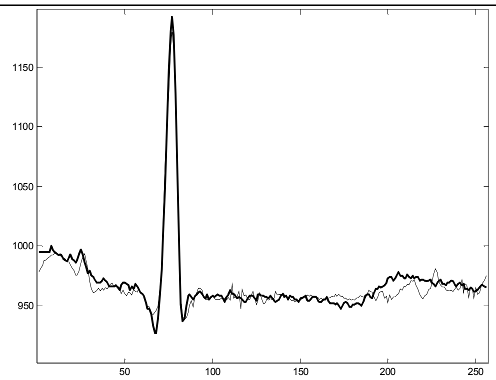
IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

Compression based on the Coiflet4 dictionary

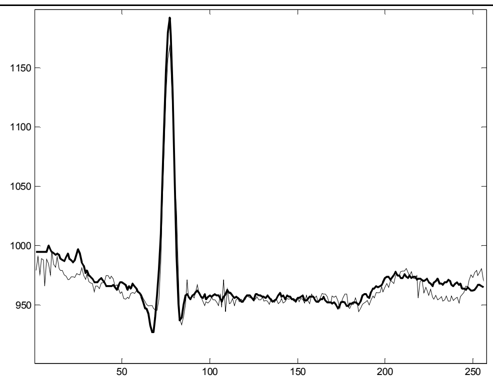
In order to test the compression performances of the Coiflet4 type wavelet overcomplete dictionary described above, the first 256 samples from the recording with number 100 have been used. The results presented in Figure 1 are the best obtained for 10 different random projection matrixes for each of the compression ratios of 4, 6, 8 and 10. It can be seen that, except for the 4:1 compression in all the other cases, both the reconstruction errors and the visual inspection show unacceptable reconstruction errors.



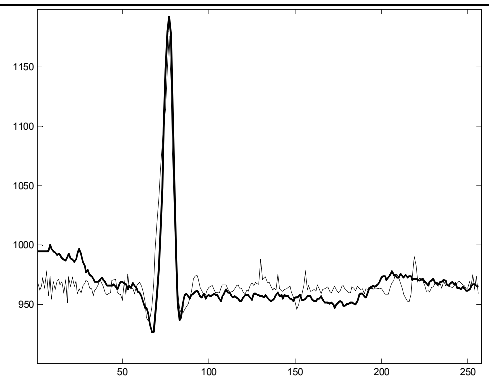
CR = 4:1; PRD = 0.582; PRDN = 17.24; RMS = 5.65



CR = 6:1; PRD = 0.80; PRDN = 23.75; RMS = 7.78



CR = 8:1; PRD = 1.02; PRDN = 30.25; RMS = 9.92



CR = 10:1; PRD = 1.51; PRDN = 44.72; RMS = 14.66

FIG. 1 –ORIGINAL (THIN LINE) AND RECONSTRUCTED (THICK LINE) ECG SIGNALS USING A COIFLET4 OVERCOMPLETE DICTIONARY.

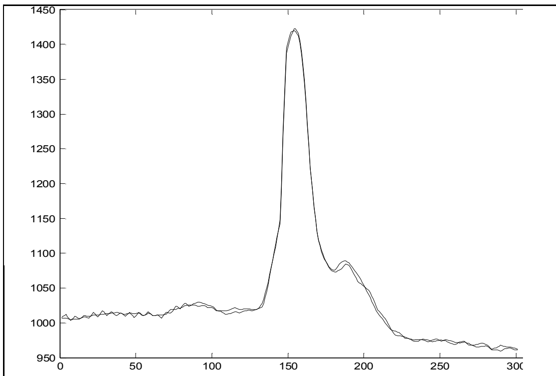
The weak results obtained with overcomplete wavelet dictionaries for CR over 4:1 have determined us to adopt the method of construction of specific dictionaries for the ECG signal based on cardiac patterns.

Compression and classification using RCP and RRCP

The testing of the compression method has been made on 1000 cardiac patterns taken from the 24 ECG recordings, evenly distributed for all 8 classes. The two previously presented types of dictionaries together with the two types of projection matrixes have been used. In order to evaluate the compression we computed the CR, the distortion, the QS [10], the classification ratio of the cardiac patterns and the confusion matrix.

Together with compression, for the case of dictionaries constructed with cardiac patterns (RCP and RRCP) we have also investigated the classification possibility of using the largest coefficient α necessary for the reconstruction obtained using the BP technique as a result of the CS technique. This classification mode is justified to a certain extent by the fact that the reconstructed ECG wave is a linear combination of patterns from the dictionary, the greater weighting being expected for the component which resembles most the ECG signal and carrying significant information about the pathology.

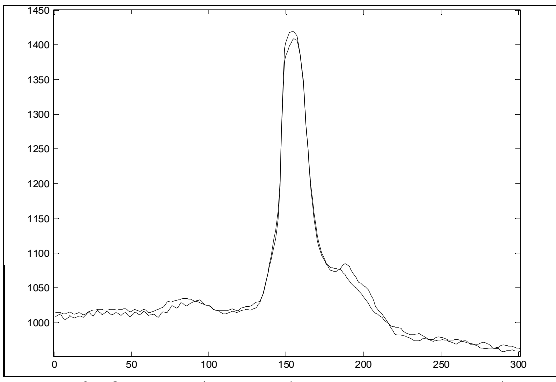
We present in Fig. 2 presents the results for a RRCP dictionary with {0,1} random projection matrix respectively pseudo-random projection matrix.



	class 1	class 2	class 3	class 4	class 5	class 6	class 7	class 8
class 1	**69.23**	0.0	7.69	0.0	7.69	7.69	7.69	0.0
class 2	15.38	**76.92**	0.0	0.0	0.0	0.0	7.69	0.0
class 3	9.09	9.09	**81.81**	0.0	0.0	0.0	0.0	0.0
class 4	7.69	7.69	0.0	**84.61**	0.0	0.0	0.0	0.0
class 5	0.0	0.0	0.0	0.0	**100**	0.0	0.0	0.0
class 6	21.42	0.0	0.0	0.0	0.0	**78.57**	0.0	0.0
class 7	0.0	0.0	0.0	0.0	0.0	0.0	**100**	0.0
class 8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	**100**

RRCP with {0,1} random matrix:

CR = 15:1 ; classification _rate = 80%; PRD_mean = 1.04 ; PRDN_mean = 15.46; RMS_mean = 1.2694e-004



	class1	class2	class3	class4	class5	class6	class7	class8
class1	**80,0**	0,0	10,0	0,0	10,0	10,0	0,0	0,0
class2	12,5	**75,0**	0,0	0,0	6,25	6,25	0,0	0,0
class3	11,11	0,0	**88,88**	0,0	0,0	0,0	0,0	0,0
class4	7,14	0,0	0,0	**85,71**	0,0	7,14	0,0	0,0
class5	12,5	0.0	12,5	0.0	**75**	0.0	0.0	0.0
class6	23,52	0.0	0.0	0.0	0.0	**64,70**	11,76	0.0
class7	0.0	40	0.0	0.0	0.0	0.0	**60**	0.0
class8	6,25	0.0	0.0	0.0	0.0	0.0	6,25	**87,5**

RRCP with pseudo random matrix:
CR = 15:1; classification _rate = 77%; PRD_mean = 1.18; PRDN_mean = 17.42; RMS_mean = 11.51

FIG. 2 - ORIGINAL (THIN LINE) AND RECONSTRUCTED (THICK LINE) ECG SIGNALS AND CONFUSION MATRICES FOR THE RRCP DICTIONARY WITH {0,1} RANDOM AND PSEUDO-RANDOM PROJECTION MATRICES RESPECTIVELY.

The compression results for the RRCPand RCP dictionaries using random projection matrixes and random 0 and 1 projection matrixes, for a 15:1 and a 20:1 compression are summarized in Tables 1 and 2.

Comments regarding compression

It can be observed, that the best compression results were obtained using the RRCPdictionary followed the RCP dictionary, both built from cardiac patterns. The above two dictionaries, being more specific and adequate to the problem significantly outperform the overcomplete Coiflet4 wavelet dictionary.

A somehow surprising but highly beneficial from the computing point of view is the fact that the compression results using random projection matrixes which contain only values of 0 and 1 are superior compared with the case random matrixes.

TABLE 1 –COMPRESSION RESULTS FOR THE RANDOM PROJECTION MATRIXES

Dictionar	CR	classification _rate	QS	PRDN_mean	PRD_mean
RRCP	15:1	77%	12,71	17.42	1.18
RCP	15:1	74%	10,13	23.02	1.48
RRCP	20:1	62%	12,5	24.37	1.60
RCP	20:1	70%	11,11	28.90	1.80

TABLE 2 –COMPRESSION RESULTS FOR THE RANDOM PROJECTION MATRIXES WITH 0 AND 1

Dictionar	CR	classification _rate	QS	PRDN_mean	PRD_mean
RRCP	**15:1**	**80%**	**14,42**	**15.40**	**1.05**
RCP	15:1	74%	10,20	23.32	1.47
RRCP	**20:1**	**67%**	**13,51**	**22.90**	**1.48**
RCP	20:1	70%	10,41	29.87	1.92

As it can be observed from the table above the results obtained with the (RRCP) dictionary with the R wave centered by resampling and with 0-1 random projection matrix compares positively regarding the PRD, RMS errors, average CR and QS with other compression results presented in the literature.

TABLE 3 - COMPARISON BETWEEN THE PROPOSED METHOD AND OTHER COMPRESSION ALGORITHMS FOR AVERAGE VALUES FOR 24 RECORDS

Algorithm	Average of errors (PRD or RMS)	Average of CR	QS
Wavelet [31]	18.2 RMS	21.4:1	
SPHIT [32]	3.57 PRD	12:1	3.39
	4.85 PRD	16:1	3.29
	6.49 PRD	20:1	3.08
JPEG2000 [33]	2.19 PRD	12:1	5.47
	2.74 PRD	16:1	5.8
	3.26 PRD	20:1	6.1
QLV – Skeleton – Huffman* [34]	0.641 PRD*	16.9:1*	29.36*
Skeleton – [10]	1.17 PRD 11.35 RMS	18.27:1	15.61
Proposed - *RRCP*	**1.04**	**15:1**	**14.42**
Proposed - *RRCP*	**1.48**	**20:1**	**13.51**

NOTE: The results reported in [34] marked with * in the Table X were obtained using a combined ECG compression method consisting of a preprocessing stage with quad level vector (QLV) for the extraction of the ECG skeleton achieving a 8.4:1 compression and a coding block (consisting of delta and Huffman Coding). The results referenced in Table 3 are the final one improved by the Huffman coding stage.

Comments regarding classification

The main purpose of this paper was to investigate aspects the possibility of compressing ECG signals based on the theory of the CS. The very simple classification of the decoded cardiac patterns based only on the largest α coefficient required for the reconstruction was an aspect which came together with the main research, was very easy to reveal and seems to be worthwhile for future research. Except for being associated to CS and extremely simple it cannot be used in this form for medical diagnostic and cannot compete with state of the art results: for 5 classes of pathologies out of 15, De Chazal [35] reports a classification accuracy of 97.4%, while Prasad [36] and Osowski [37] using wavelet and SVM respectively, report 96%.

V. CONCLUSION

The paper presents several results concerning ECG medical signals compression based on recent results on compressed sensing. The ECG signal has been compressed using a number of projections on a random matrix and reconstructed with the Basis Pursuit algorithm. Thus, knowing the ECG signal, the random projection matrix used for coding, and the dictionary which ensures the ECG signals sparsity, the coefficients necessary for reconstruction were obtained from the projections using the Interior Point Method (IPM) linear programming algorithm.

Three types of dictionaries have been used, namely, overcomplete Coiflet4 wavelet dictionaries and two specific dictionaries built from cardiac patterns. The results obtained in the case of using the Coiflet4 wavelet dictionary have shown that for compressions higher than 4:1, the reconstruction errors are unacceptable. In the case of using dictionaries constructed from cardiac patterns with the R wave centered by resampling (RRCP), compression ratios of 15:1 with an error PRD_mean of 1.04 and of 20:1 with error PRD_mean of 1.48 have been obtained. As a “byproduct”, an extremely simple pattern classification with 80% and 67% accuracy respectively has been also obtained. We have thus shown that the use of specific dictionaries adapted to the class compressed signal constitutes an advantage both from the point of view of the compression quality but also a very simple (but rough) classification method.

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