Beamformer for Cylindrical Conformal Array of Non-isotropic Antennas

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***Abstract*—The principal objective of this investigation is to facilitate minimum variance distortionless response (MVDR) beamforming technique for a cylindrical conformal array geometry. An array of directional radiating elements is postulated to cover a surface typical of the cylinder of an aircraft or missile. Borrowing the analysis of conformal array antennas, the authors first derive a deterministic expression that describes the beam pattern of arbitrary weighted cylindrical conformal array. Then, making use of the MVDR beamforming, we derive the beamformer for uniform linear array (ULA) of directional antennas which are different from the traditional omnidirectional elements. Thus, the pattern of a directional element is synthesized by the antennas on the same ring array, and we design the MVDR beamformer, which uses MVDR beamforming for ULA of the synthesized pattern. To demonstrate the validity of the method, and cylinder arrays are constructed and experimental results agree well with theoretical expectations.**

***Index Terms*—Airborne radar, Antenna arrays, Phased arrays, Antenna radiation patterns, Radar antennas.**

I. INTRODUCTION

Analysis and design different kinds of conformal arrays have been of interest for many years. In many applications of radar, sonar, biomedical imaging, and wireless communications, conformal array has been known for a long while to be able to give a large angular coverage with good radiation pattern characteristics, reduction of aerodynamic drag, space savings, and potential increase in available aperture and reduction or elimination of radome included boresight errors. We can get a great deal of information on conformal array in [1]-[3]. So far, in practice, conformal array has been restricted to antennas with moderate number of elements, or to very specific applications, for example telemetry for missile.

Up to now, many researchers around the world have studied the conformal array, such as pattern synthesis [4]-[5], superresolution with conformal array [6], and estimation of direction-of-arrival [7]. Due to the number of conformal array antenna is large and the location of its elements is irregular, research about the beamforming for conformal array has not been exploited much in the existing literature, and presents new challenges.

Beamforming is one such technique that is the use of adaptive or smart antennas to produce a movable beam pattern, which can be directed to the desired coverage areas and minimize the impact of unwanted noise and interference, thereby improving the quality of desired signal. Conventional approaches of Beamforming for ULA are relatively mature, such as least mean square(LMS) algorithm[8], [9], recursive least square (RLS) [10], [11] algorithm, MVDR [12]-[14] algorithm, and other combined algorithms[15]-[17]. The LMS or RLS are two commonly used algorithms for beamforming. The former has good tracking performance with low computational complexity, and is robust against numerical errors [8]. On the other hand, the RLS algorithm can achieve a faster convergence that is independent of the engine-value spread variations of the covariance matrix [10]. MVDR beamformer is one of the key algorithms in signal processing. Experiments showed that MVDR and other existing algorithms perform well when omnidirectional elements are used, but do not provide much enhancement when directional elements are used. We will show that MVDR can be used for directional elements as well.

In this paper, beamformer we present is different from previous methods, and it can be used for element which has arbitrary directional response. The proposed MVDR beamformer is applied to and cylindrical arrays in Section V. The optimum beam patterns in our experiment show that the proposed approach indeed has deep notches at the locations of interferers, and creates the main lobe at the direction of the expected signal.

II. CYLINDRICAL CONFORMAL ARRAY SIGNAL MODEL

In this section we consider a $D \times E$ cylindrical conformal array shown in Fig.1.

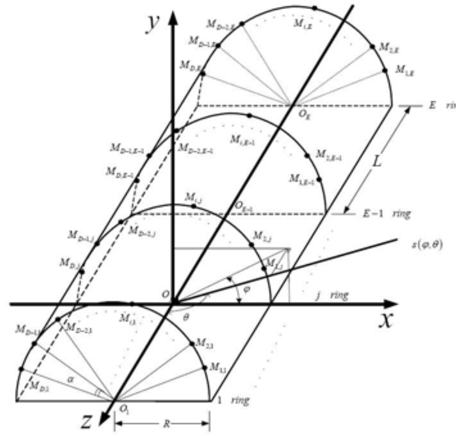

Figure 1. Geometry and global coordinate system of $D \times E$ cylindrical conformal array.

There are D directional elements uniformly distributed in each ring with the included angle α and E rings with

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equal spacing L on the cylindrical conformal array of radius R. The global coordinate system is assigned to the j th ring planar array. We assume that there is a narrowband source $s(\varphi,\theta)$ in the far field of the array with azimuth angle $\varphi(0\le\varphi\le\pi)$ and elevation angle $\theta(0\le\theta\le\pi)$ impinging on the antenna array.

In ULA, the array factor and the element pattern are separable, because it is the case of a planar array. In contrast, this condition no longer holds for the case of a general conformal array. Cylindrical conformal array pattern function can not be defined such that the far field is the product of the element radiation pattern and the composite array pattern function. Hence, signals must be evaluated at the element level.

We define the wavenumber $\mathbf{k}$ as

$$\mathbf{k}=-\frac{2\pi}{\lambda}\begin{bmatrix}\sin\theta\cos\varphi\\ \sin\theta\sin\varphi\\ \cos\theta\end{bmatrix} \tag{1}$$

where λ is the wavelength corresponding to the frequency of signal.

The far field of a system of simultaneously excited cylindrical conformal array of antennas may be written, omitting the time phase factor $e^{j\omega t}$, as

$$F(\varphi,\theta)=\sum_{i=1}^{D}\sum_{j=1}^{E}\left[a_{i,j}^{*}f\left(\varphi_{i,j},\theta_{i,j}\right)e^{-\mathbf{k}^{T}\mathbf{p}_{i,j}}\right] \tag{2}$$

where φ and θ are azimuth angle and elevation angle in the global coordinate system, $a_{i,j}$ is the complex coefficient of the radiating element at the i th generating line and j th ring, $a_{i,j}^{*}$ is the conjugate of $a_{i,j}$. Element pattern function $f\left(\varphi_{i,j},\theta_{i,j}\right)$ is designed and determined by $\varphi_{i,j}$ and $\theta_{i,j}$ in their local coordinate system. ($\varphi_{i,j}$ and $\theta_{i,j}$ are azimuth angle and elevation angle in their local coordinates accordingly.) It can be known from Fig.2 that elements along the same generating line have the same element pattern function, so $f\left(\varphi_{i,j},\theta_{i,j}\right)$ can be written as $f\left(\varphi_{i},\theta_{i}\right)$ $(1\le i\le D)$. $\mathbf{p}_{i,j}=\left(R\cos\varphi_i,R\sin\varphi_i,z_j\right)^{T}$ is the position of the element in the global coordinate system. $(\cdot)^{T}$ denotes the transpose of a matrix or vector. The j th ring cut of cylindrical conformal is depicted in Fig.2. We assume that $f\left(\varphi_i,\theta_i\right)=\cos\theta_i$.

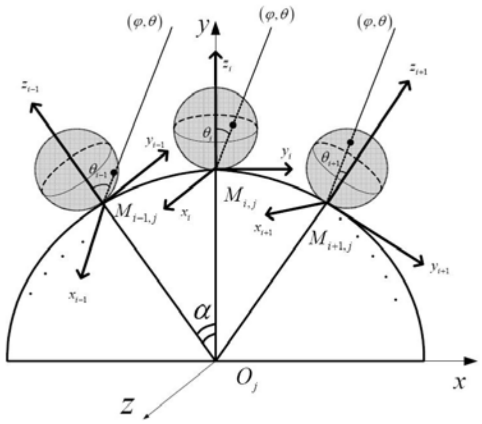


Figure 2. The j th ring cut of cylindrical conformal array

From (2), we know that $f\left(\varphi_i,\theta_i\right)$ is determined by φ_i and θ_i, so local coordinate system $\left(x_i,y_i,z_i\right)$ is assigned to every element by right-hand coordinate system: the z_i-axis is the surface normal and x_iy_i -plane is the corresponding tangent plane of carrier surface.

The relationships between rectangular and spherical coordinates is

$$\begin{cases}x=\rho\sin\theta\cos\varphi\\ y=\rho\sin\theta\sin\varphi\\ z=\rho\cos\theta\end{cases}\qquad\begin{cases}x_i=\rho_i\sin\theta_i\cos\varphi_i\\ y_i=\rho_i\sin\theta_i\sin\varphi_i\\ z_i=\rho_i\cos\theta_i\end{cases} \tag{3}$$

From Fig.2, if we assume the z_i-axis of elements along the i th generating line and the y -axis of the global coordinate system are the same (like $i=D/2$ or $i=(D\pm1)/2$). The relationships between the global coordinate system (x,y,z) and the local coordinate system $\left(x_{i+1},y_{i+1},z_{i+1}\right)$ of the $(i+1)$ th generating line is:

$$\begin{aligned}\begin{bmatrix}x\\ y\\ z\end{bmatrix}&=R_1R_2R_3\begin{bmatrix}x_i\\ y_i\\ z_i\end{bmatrix}\\ &=\begin{bmatrix}0&1&0\\ -1&0&0\\ 0&0&1\end{bmatrix}\begin{bmatrix}0&0&-1\\ 0&1&0\\ 1&0&0\end{bmatrix}\begin{bmatrix}1&0&0\\ 0&\cos\alpha&\sin\alpha\\ 0&-\sin\alpha&\cos\alpha\end{bmatrix}\begin{bmatrix}x_i\\ y_i\\ z_i\end{bmatrix}\end{aligned} \tag{4}$$

where R_1, R_2 and R_3 are three Euler rotation matrices. Through (4), we can get the transformation from (φ,θ) to $\left(\varphi_{i+1},\theta_{i+1}\right)$ is realized as follows:

$$\begin{cases}\varphi_{i+1}=\arccos\left[\dfrac{\cos\theta}{\sqrt{1-\cos^{2}\left(\varphi_n-\varphi\right)\sin^{2}\theta}}\right]\\ \theta_{i+1}=\arccos\left[\cos\left(\varphi_n-\varphi\right)\sin\theta\right]\end{cases} \tag{5}$$

where φ_n is the azimuth angle of $\mathbf{p}_{i,j}$.

Deducing from the equations in (5), we can rewrite (2) as

$$\begin{aligned}F(\varphi,\theta)&=\sum_{i=1}^{D}\sum_{j=1}^{E}\left\{a_{i,j}^{*}f_i(\varphi,\theta)e^{-\mathbf{k}^{T}\mathbf{p}_{i,j}}\right\}\\ &=\sum_{i=1}^{D}\sum_{j=1}^{E}\left\{a_{i,j}^{*}f_i(\varphi,\theta)e^{-\frac{2\pi}{\lambda}\left[R\sin\theta\cos\left(\varphi-\varphi_n\right)+z_n\cos\theta\right]}\right\}\end{aligned} \tag{6}$$

where $f_i(\varphi,\theta)$ is the pattern function of the elements along the same generating line. Now, $F(\varphi,\theta)$ is directly related to φ and θ of arriving signal.

III. MVDR BAMFORMING FOR ULA OF DIRECTIONAL ELEMENTS

Consider a linear array consisting of M antennas. Assuming that K arriving signals are narrowband signals, we denote the array steering vector as

$$\mathbf{V}\left(\theta_k\right)=\begin{bmatrix}1 & e^{-j\phi_k} & \cdots & e^{-j(M-1)\phi_k}\end{bmatrix}^{T} \tag{7}$$

where ϕ_k is the phase delay due to propagation and can be represented as

$$\phi_k=\frac{2\pi d_k}{\lambda}\sin\theta_k \qquad 1\le k\le K \tag{8}$$

where d_k is the position of the k th element. and signal impinges on the array at an angle θ_k.

We know the output of linear array at time n is given by

$$y(n) = \mathbf{w}^H \mathbf{x}(n) \tag{9}$$

where $\mathbf{x}(n) = \left[x_1(n) \ \cdots \ x_M(n)\right]^T$ is the array observation vector, $\mathbf{w} = \left[w_1(n) \ \cdots \ w_M(n)\right]^T$ is the complex vector of beamformer weights. and $(\cdot)^H$ denotes the Hermitian transpose.

The observation vector can be written as

$$\begin{aligned}\mathbf{x}(n) &= \mathbf{s}(n) + \mathbf{i}(n) + \mathbf{v}(n) \\ &= s(n)\mathbf{V}(\theta_1) + \mathbf{i}(n) + \mathbf{v}(n)\end{aligned} \tag{10}$$

where $\mathbf{s}(n)$, $\mathbf{i}(n)$, and $\mathbf{v}(n)$ are the desired signal, interference, and noise components, respectively. $\mathbf{V}(\theta_1)$ is the presumed desired signal steering vector. $s(n)$ is the desired signal waveform. The desired signal and interferers are assumed to be uncorrelated.

(10) can also be written as

$$\mathbf{x}(n) = \mathbf{A}\underline{\mathbf{s}}(n) + \mathbf{v}(n) \tag{11}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\phi_1} & e^{-j\phi_2} & \cdots & e^{-j\phi_K} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j(M-1)\phi_1} & e^{-j(M-1)\phi_2} & \cdots & e^{-j(M-1)\phi_K} \end{bmatrix}$$
$$= \left[\mathbf{V}(\theta_1) \ \ \mathbf{V}(\theta_2) \ \ \cdots \ \ \mathbf{V}(\theta_K)\right]$$

and $\underline{\mathbf{s}}(n)$ includes both $\mathbf{s}(n)$ and $\mathbf{i}(n)$.

The MVDR beamformer [18] minimizes the output interference-plus-noise power while maintaining a distortionless response to the desired signal. The MVDR problem is given by

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \qquad st. \ \mathbf{w}^H \mathbf{V}(\theta_1) = 1 \tag{12}$$

where

$$\mathbf{R} = E\left[\mathbf{x}(n)\mathbf{x}^H(n)\right]$$

is $M \times M$ correlation matrix, and $E[\cdot]$ denotes the expectation operator. The solution to the MVDR beamforming problem (9) is given by [18]

$$\mathbf{w}_{opt} = \frac{R^{-1}\mathbf{V}(\theta_1)}{\mathbf{V}^H(\theta_1)R^{-1}\mathbf{V}(\theta_1)} \tag{13}$$

In many applications, the elements have a non-isotropic beam pattern function $f(\theta_k)$. As in the ring array case, the main response axis of the element pattern will point in a radial direction. Rewriting (11) to show the difference above

$$\tilde{\mathbf{x}}(n) = \tilde{\mathbf{A}} \, \underline{\mathbf{s}}(n) + \mathbf{v}(n) \tag{14}$$

where

$$\tilde{\mathbf{A}} = \left[f(\theta_1)\mathbf{V}(\theta_1) \ \cdots \ f(\theta_K)\mathbf{V}(\theta_K)\right]$$
$$= \begin{bmatrix} f(\theta_1) & \cdots & f(\theta_K) \\ f(\theta_1)e^{-j\phi_1} & \cdots & f(\theta_K)e^{-j\phi_K} \\ \vdots & \ddots & \vdots \\ f(\theta_1)e^{-j(M-1)\phi_1} & \cdots & f(\theta_K)e^{-j(M-1)\phi_K} \end{bmatrix}$$

is determined by both scale factor $f(\theta_k)$ and $\mathbf{V}(\theta_k)$.

So

$$\begin{aligned}\tilde{\mathbf{R}} &= E\left[\tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^H(n)\right] \\ &= \tilde{\mathbf{A}}E\left[\underline{\mathbf{s}}(n)\underline{\mathbf{s}}(n)^H(n)\right]\tilde{\mathbf{A}}^H + E\left[\mathbf{v}(n)\mathbf{v}^H(n)\right]\end{aligned}$$

In many cases, $\tilde{\mathbf{A}}$ satisfies the conditions that $f(\theta_k) \neq 0 \ \ (1 \le k \le K)$ and the family of vectors $f(\theta_k)\mathbf{V}(\theta_k) \ \ (1 \le k \le K)$ is linearly independent. $\tilde{\mathbf{R}}$ is positive definite matrix all the same.

Now, we can describe the MVDR beamforming problem in ULA as

$$\min_{\tilde{\mathbf{w}}} \tilde{\mathbf{w}}^H \tilde{\mathbf{R}} \tilde{\mathbf{w}} \qquad st. \ \tilde{\mathbf{w}}^H \tilde{\mathbf{V}}(\theta_1) = 1 \tag{15}$$

where $\tilde{\mathbf{w}}$ s the complex vector of beamformer weights in ULA of directional elements, $\tilde{\mathbf{V}}(\theta_1) = f(\theta_1)\mathbf{V}(\theta_1)$.

the closed-form solution to (15) is

$$\tilde{\mathbf{w}}_{opt} = \frac{\tilde{R}^{-1}\mathbf{V}(\theta_1)}{f^*(\theta_1)\mathbf{V}^H(\theta_1)\tilde{R}^{-1}\mathbf{V}(\theta_1)} \tag{16}$$

IV. Numerical Validation

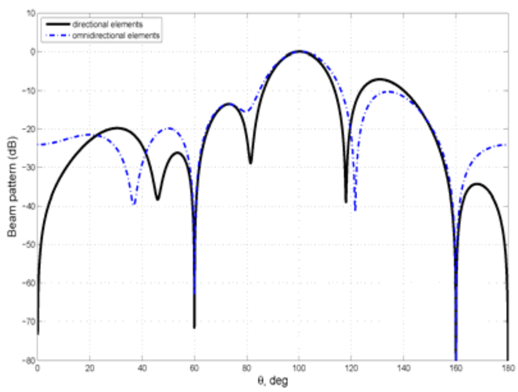
To derive the beamformer, we will first get the quiescent beam patterns of 4×1 and 5×1 cylindrical conformal arrays respectively, and then synthesize patterns of the elements in the same ring, thus consider the synthesized pattern as a directional element in ULA. later, we will distribute the beamformer weights obtained in ULA to every antenna of cylindrical conformal array to achieve the beamformer for cylindrical array.

The simulation setup is as follows: uniform linear array with 4 elements spaced at a half wavelength apart. $\alpha = 9^\circ$, $\lambda = 0.01/3\text{m}$, $L = \lambda/2$, $R = L/\alpha$. The additive noise plus interference is assumed to be a zero mean complex Gaussian noise. We use 512 snapshots.

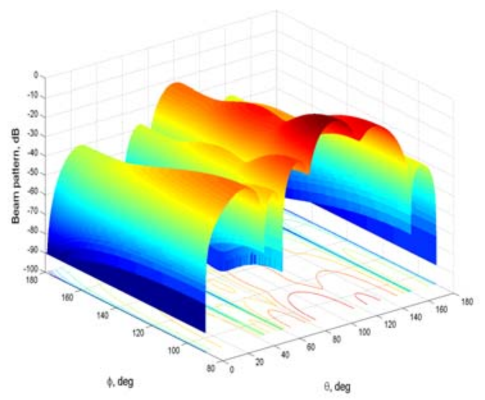
There are three signals impinging upon all arrays:

1) the signal of interest $s(t)$ with angle of arrival $\theta_1 = 100^\circ$,
2) an interference signal $i_1(t)$ with angle of arrival $\theta_{\text{int}1} = 60^\circ$ and $\sigma_{i\text{nt}1}^2 = 10^4$ (40dB above noise),
3) another interference signal $i_2(t)$ with angle of arrival $\theta_{\text{int}2} = 160^\circ$ and $\sigma_{i\text{nt}1}^2 = 10^4$ (40dB above noise).

The received noise variance $\sigma^2 = 1$ and the amplitude of $s(t), i_1(t)$, and $i_2(t)$ is determined based on the SNR and INR.

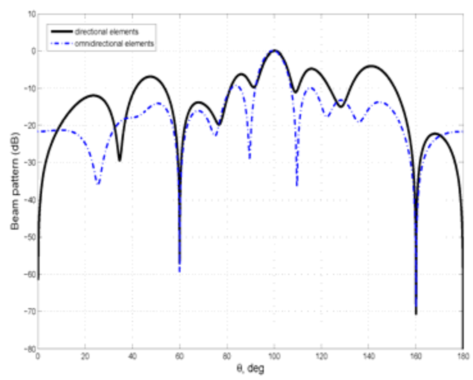


(a)

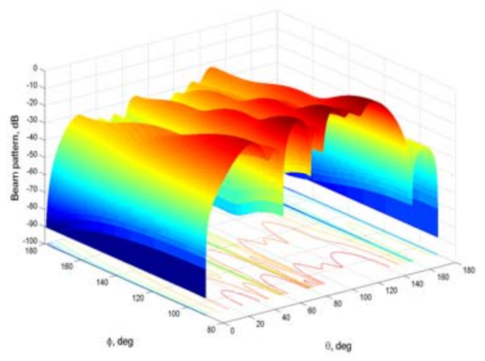


(b)

Figure 3. (a) MVDR beamformer: beam pattern for ULA of 6 elements and 4×6 cylindrical conformal array cut for $\varphi = 90^{\circ}$;
(b) Beam pattern for 4×6 cylindrical conformal array and contour plot of magnitude of beam pattern.



(a)



(b)

Figure 4. (a) MVDR beamformer: beam pattern for ULA of 11 elements and 5×11 cylindrical conformal array cut for $\varphi = 90^{\circ}$;
(b) Beam pattern for 5×11 cylindrical conformal array and contour plot of magnitude of beam pattern.

The simulation results are shown in Fig. 2 and 3. It can be seen that in the sidelobe region the optimum beam pattern has a deep notch at the locations of interferers and the other lobes are well behaved. The MVDR beamformer for cylindrical array outperforms the ULA of omnidirectional elements in terms of the main beamwidth, but the sidelobe level is not improved noticeably. In contrast, sidelobe structure in elevation shows differences; the reason is that the directional element has different gain as the angle changes. At the locations of interferers, the depth of the directional elements is about -70dB, which is deeper than the omnidirectional. The beamformer of directional elements converges more rapidly if the jammer arrives in that area.

V. Conclusion

The beamformer for cylindrical conformal array is discussed in this paper. A synthesis method based on directional pattern for ULA is used for beamforming of cylindrical conformal array. The combination of the proposed synthesis pattern method and the MVDR algorithm for ULA has established a effective method for uniform spaced cylindrical conformal array. Experiments indicate that this method is an effective tool for practical conformal array beamforming and it can be used in antenna for short range gigabit wireless communication at millimetre-wave frequencies. Simulation results are provided to demonstrate the effectiveness and behavior of the proposed method.

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