A Novel Fault Identification Using WAMS/PMU

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***Abstract*—The important premise of the novel adaptive backup protection based on wide area information is to identify the fault in a real-time and on-line way. In this paper, the principal components analysis theory is introduced into the field of fault detection to locate precisely the fault by mean of the voltage and current phasor data from the PMUs. Massive simulation experiments have fully proven that the fault identification can be performed successfully by principal component analysis and calculation. Our researches indicate that the variable with the biggest coefficient in principal component usually corresponds to the fault. Under the influence of noise, the results are still accurate and reliable. So, the principal components fault identification has strong anti-interference ability and great redundancy.**

***Index Terms*—Fault identification, Noise, Principal component analysis, Wide area measurement system, WAMS.**

I. Introduction

Under the competitive and deregulated environments during recent years, the complexity of power systems has become more and more high. As a result, how to implement the better and advanced security monitor and control of power system has also become an increasing important problem, especially for the power system relay protections. Though the backup protection in the protective system is still playing an important role, the coordination of these protections is to be confronted with the great difficulty. It is a matter of fact that about 75% of the large-scale power cascading outages worldwide have some concerns of the backup protection [1]. The traditional backup protection based only on the local information even adds fuel to the flames in the process of some accidents. With the increasing installation of phasor measurement units (PMUs) and the implementation of wide area measurement system (WAMS) [2]–[6], it is able to provide phasor measurement information, including nodal voltage phasors and branch current phasors, in the whole electric power system synchronically. The phasor information reflects both realistically and objectively the operation condition of the current system and provides a new perspective to the design of backup protection.

When a disturbance happens in the power system, the voltage and current information in the whole network will change accordingly. Especially, the monitoring and the control of oscillation have become an important direction to the application of WAMS. As to relay protection, how to extract the fault features and to detect the fault from the large amount of measurement information supplied by the WAMS is the key problem for the backup protection based on the identification of fault [7]–[9]. Different to the traditional fault diagnose methods relying on the information of circuit breakers statues change and primary or backup protection trip signals, the realization of novel backup protection has the potential requirement for real-time and online identification of the fault, in which the new mathematical method should be introduced to deal with the synchronical vector information in the overall system.

Principal component analysis (PCA) [10]–[13] is a linear technique for mapping a multi-dimensional data set into a lower dimension space while minimizing the loss of information. It is an important and essential technique for data reduction, image compression, and feature extraction. And it has been widely used in many fields including data communication, pattern recognition, and image processing. In the present paper, the fault identification based on WAMS will be discussed carefully. The PCA method will be introduced into the field of fault detection to locate precisely the fault by mean of the voltage and current phasor data from the PMUs. Considering the influence of noise, we will extract the distinct features of faults.

II. A Brief Introduction of Wide Area Measurement System

WAMS, Wide Area Measurement System, can obtain phasor information on node voltage and branch current with uniform time scale and monitor the running state of the current power system. Phasor measurement unit is the remote measurement devices of the WAMS, which is the product of the wide application of Global Position System (GPS) in the world. In 1980s, Professor Arun G. Phadke and James S. Throp created the first PMU equipment in Virginia Tech in USA [2]. Except the GPS receiver, the basic structure and principle of PMUs is very similar to a computer relay. PMUs dispersedly equipped in the electric power system could obtain the same sampling clock by utilizing the synchronized clock signals from GPS. And the corresponding input signals (consisting of nodal voltages and feeder currents) will be sampled and converted into positive sequence quantities, negative sequence quantities and zero sequence quantities. Consequently, the operation condition of the power system in one snapshot is to be depicted with mutil-points synchronized phasors indicating in the same coordinate.

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Fig. 1 presents a typical structure in PMU [2]. Compare to the traditional measurement system such as Supervisory Control and Data Acquisition (SCADA)/Energy Management System (EMS) with Remote Terminal Unit (RTU), the modern measurement system PMU/WAMS not only could implement the functions required in the conventional measurement system, but also will or has brought profound impact on state estimation, dynamic monitoring and system protection and so on.

The architecture of WAMS could be divided as different levels. In each level, the Phasor Data Concentrator (PDC) could match the time tags of data received from the various PMUs. Then the phasor data stream will be created for application, and communicated to upper levels [14]. In this structure system, different level will take on various functions. Especially, in the researches of the authors, the regional or central control centers will be the appropriate target levels to implement the wide area backup protection, which requires the phasor data from much wider areas, even the whole system, with the longer time delay. A classical architecture of the WAMS has been shown in Fig. 2.

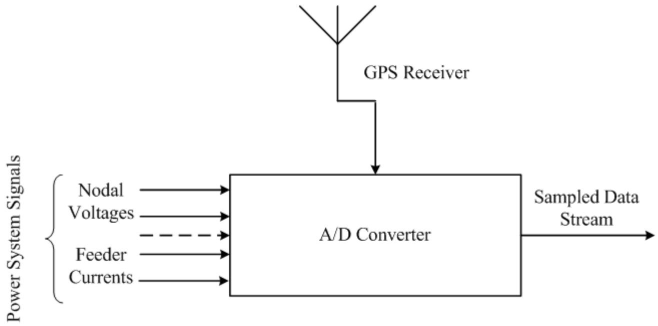

Figure 1. A simple depiction of the phasor measurement unit

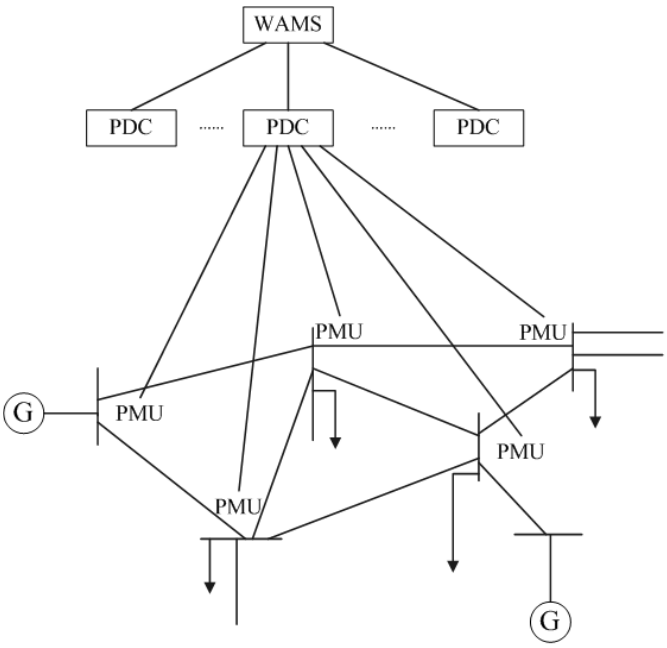

Figure 2. A typical hierarchical structure of wide area measurement system

III. Principal Component Analysis

Principal component analysis is concerned with explaining the variance-covariance structure of a set of variables through a few linear combinations of these variables. Its general objectives are [15]–[17]:

- Dimension reduction;
- Interpretation variables;
- Identifying patterns of association among variables.

Generally, the principle of PCA is explained as follows.

Let $x=(x_1,x_2,\cdots,x_p)'$ be p-dimensional random variable. One assumes its corresponding second-order moments exist. In practical data acquired by PMU, the column represents collecting time of PMU, and the corresponding raw is the sychro-phasor information of current and voltage measured by PMU in real-time. The mean vector and covariance matrix of X are:

$$\mu=E(X),\quad \Sigma=D(X)\,. \tag{1}$$

Consider linear combinations,

$$\begin{cases} f_1=a_{11}x_1+a_{12}x_2+\cdots+a_{1p}x_p=a_1'x \\ f_2=a_{21}x_1+a_{22}x_2+\cdots+a_{2p}x_p=a_2'x \\ \quad\vdots \\ f_p=a_{p1}x_1+a_{p2}x_2+\cdots+a_{pp}x_p=a_p'x \end{cases} \tag{2}$$

or it can be expressed as

$$f=A'x \tag{3}$$

wherein $f=(f_1,f_2,\cdots,f_p)',\quad A=(a_1,a_2,\cdots,a_p)$.

One hopes to find out a set of new uncorrelated variables $f_1,f_2,\cdots,f_m\ (m\le p)$, which can fully reflect the information of the original variables $x_1,x_2,\cdots,x_p$. Meanwhile, for $f_1,f_2,\cdots,f_m$, one has the following relationships,

$$\begin{aligned} D(f_i)&=D(a_i'x)\\ &=a_i'D(x)a_i''=a_i'\Sigma a_i\\ &(i=1,2,\cdots,m)\end{aligned} \tag{4}$$

and

$$\begin{aligned} \mathrm{cov}(f_i,f_k)&=\mathrm{cov}(a_i'x,a_k'x)\\ &=a_i'\,\mathrm{cov}(x,x)a_k''=a_i'\Sigma a_k\\ &(i,k=1,2,\cdots,m)\end{aligned} \tag{5}$$

Then, the problem to search for new variables can be transformed into another one; that is, under the condition of $f_1,f_2,\cdots,f_m$ are mutually independent, one can solve a_i which will maximize $D(f_i)=a_i'\Sigma a_i$. One therefore defines,

The first principal component is $f_1=a_1'x$ that maximizes $D(f_1)=a_1'\Sigma a_1$ subject to $a_1'a_1=1$.

The second principal component is $f_2=a_2'x$ that maximizes $D(f_2)=a_2'\Sigma a_2$ subject to $a_2'a_2=1$ and $\mathrm{cov}(f_2,f_1)=\mathrm{cov}(a_2'x,a_1'x)=0$.

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The kth principal component is $f_k=a_k'x$ that maximizes $D(f_k)=a_k'\Sigma a_k$ subject to $a_k'a_k=1$ and $\mathrm{cov}(f_k,f_i)=\mathrm{cov}(a_k'x,a_i'x)=0\ \ (i<k)$.

Specifically, in order to get the first principal component, an objective function needs to be constructed,

$$\varphi_1(a_1,\lambda)=a_1'\Sigma a_1-\lambda(a_1'a_1-1) \tag{6}$$

Let us differentiate it,

$$\frac{\partial \varphi_1}{\partial a_1} = 2\Sigma a_1 - 2\lambda a_1 = 0 \tag{7}$$

that is,

$$(\Sigma - \lambda \mathrm{I}) a_1 = 0 \tag{8}$$

Furthermore, left multiplication $a_1^{'}$, one can get

$$a_1^{'} \Sigma a_1 = \lambda \tag{9}$$

In conclusion, the covariance matrix of $x = (x_1, x_2, \cdots, x_p)^{'}$ is Σ, and its characteristic roots are $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p \geq 0$, their corresponding unitization characteristic vectors are in sequence of $a_1, a_2, \cdots, a_p$. Therefore, the principal components are respectively

$$f_1 = a_1^{'} x,\ f_2 = a_2^{'} x, \cdots,\ f_m = a_m^{'} x \tag{10}$$

IV. Fault Identification Based on Wide Area Measurements

Principal component analysis is one of the most useful technologies for screening multivariate data. It is helpful whenever we want to group experimental objects into subgroups of similar types. It allows us to re-express the data so that the first few resulting new variables account for as much of the available information as possible. It allows the researcher to reorient the data so that the first few dimensions account for as much of the available information as possible. If there is substantial redundancy present in the data set, then it may be possible to account for most of the information in the original data set with a relatively small number of dimensions. Generally speaking, the principal components have three characteristics [17]:

- They are uncorrelated;
- The first principal component accounts for as much of the variability in the data as possible;

Each succeeding component accounts for as much of the remaining variability as is possible.

To power system, in different time for collecting electric quantities before and after some faults, the global information uploaded by PMU can be equivalent to information vector with specific dimensions, that is, the voltage vector of each node at the same time can constitute a voltage acquisition vector with the dimension of total nodes. In the same way, the current vector of each branch at the same time can constitute a current acquisition vector with the dimension of total branches. Suppose there are n nodes, b branches in system, the voltage acquisition vector and the current acquisition vector can be expressed as,

$$\begin{aligned} U_{sample} &= (\dot{U}_{node1}, \dot{U}_{node2}, \cdots, \dot{U}_{noden})^T \\ I_{sample} &= (\dot{I}_{branch1}, \dot{I}_{branch2}, \ldots, \dot{I}_{branchb})^T \end{aligned} \tag{11}$$

In the course of concrete treatment, these vectors can be further broken down into amplitude acquisition vector and phase angle acquisition vector with sequence component form and phase component form. Take voltages as example,

$$\begin{aligned} U_{sample(ABC)}^{Mag} &= Mag(U_{sample(ABC)}) \\ U_{sample(ABC)}^{Ang} &= Ang(U_{sample(ABC)}) \end{aligned} \tag{12}$$

wherein, $Mag(\ )$ and $Ang(\ )$ represent amplitude and phase angle of the corresponding vectors respectively.

Firstly, let us consider IEEE9-Bus system, the electric diagram of IEEE 9-Bus system can refer to [9]. In the structure of electricity grid, Bus-1 appears single-phase to ground fault. By BPA simulation and program calculation with MATLAB, the vector-valued of corresponding variables is only exported one times in each period. According to the current measurement precision of PMU, suppose the standard deviation of voltage vector and current vector is 0.006, and the mean error is 0 [18]. Using these actual measurement data of corresponding variables and considering the influence of noise at this level, we can carry through fault identification of fault component.

A. Fault identification of IEEE9-Bus system based on node positive sequence voltage

For IEEE9-Bus system, we can get node positive sequence voltages at t_{-1}, t_0 (Fault) and t_1 three times. Firstly, the covariance matrix of node positive sequence voltages could be calculated. In the covariance matrix, the covariance of Bus1 is 0.054049, which is the biggest. One can analyze preliminarily that the Bus1 is a probable fault component. The eigenvalues of covariance matrix have been listed in Table I.

TABLE I. THE EIGENVALUES OF COVARIANCE MATRIX BASED ON NODE POSITIVE SEQUENCE VOLTAGE

No.	Eigenvalues	Proportion	Cumulative
1	0.15354686	0.9972	0.9972
2	0.00043502	0.0028	1.0000

The first principal component can be expressed as,

$$\begin{aligned} f_1 = {} & 0.592521 x_1 + 0.225182 x_2 + 0.246964 x_3 \\ & + 0.433424 x_4 + 0.447242 x_5 + 0.229076 x_6 \\ & + 0.231830 x_7 + 0.129255 x_8 + 0.162516 x_9 \end{aligned} \tag{13}$$

Because the cumulative of the first principal component has reached 99.72%, in this place, we only need to extract the first principal component.

Based on a comprehensive analysis of these present results, a main conclusion has been reached as followings: From the feature of the first principal component, Bus1 corresponds with variable x_1, and the coefficient of x_1 is 0.592521, which is also the biggest, so, Bus1 is just the fault component. This conclusion is entirely identical with the fault set in advance.

B. Fault identification of IEEE9-Bus system based on node negative sequence voltage

Except for the positive sequence component, PMUs can also provide other negative and zero sequence data. Having taken into account for the effect of the ground points of transformers on the zero sequence component, the negative one will be chose to make further study. Similarly, we can also get node negative sequence voltages at t_{-1}, t_0 (Fault) and t_1 three times from the simulation of BPA software. With the help of MATLAB, the covariance matrix of node negative sequence voltages can be calculated.

After the analysis of diagonal elements in the covariance matrix, the covariance value 0.037868 corresponding to the Bus1 has an obvious feature, which is the biggest one in all the diagonal elements. Hence, the Bus1 may be the fault section according to the remarkable difference from other components.

In order to obtain accurate and convincing conclusion on the fault section, it is necessary to calculate and solve the eigenvalues of the above covariance matrix, which has been shown in Table II.

TABLE II. THE EIGENVALUES OF COVARIANCE MATRIX BASED ON NODE NEGATIVE SEQUENCE VOLTAGE

No.	Eigenvalues	Proportion	Cumulative
1	0.08817267	0.9989	0.9989
2	0.00009718	0.0011	1.0000

Finally, the first principal component can be expressed as,

$$\begin{aligned} f_1 = {} & 0.655287x_1 + 0.197807x_2 + 0.209107x_3 \\ & + 0.413928x_4 + 0.446243x_5 + 0.188606x_6 \\ & + 0.236759x_7 + 0.102741x_8 + 0.122857x_9 \end{aligned} \tag{14}$$

In addition, the cumulative of the first principal component is also given as 99.89% in the same data table, which illustrates that the principal component can be extracted to finish the identification of the fault. According to the detailed expression of the first principal component shown as f_1, the section in power system related to variable x_1 has the biggest coefficient as 0.655287, which is the real location of the fault. Since the Bus1 has been set as this variable x_1 in the process of mentioned calculation, the fault component Bus1 is determined. The result is absolutely identical with the prior fault set.

Now let us further study IEEE 39-Bus system, the electric diagram of IEEE 39-Bus system can also refer to [9]. In the structure of electricity grid, Bus-18 appears three-phase short-circuit to ground fault. By BPA simulation and program calculation with MATLAB, the vector-valued of corresponding variables is only exported one times in each period. According to the current measurement precision of PMU, suppose the standard deviation of voltage vector and current vector is 0.006, and the mean error is 0. Using these actual measurement data of corresponding variables and considering the influence of noise at this level, we can carry through fault identification of fault component.

C. Fault identification of IEEE 39-Bus system based on node positive sequence voltage

Likewise, we calculate the node positive sequence voltage at t_{-1}, t_0 (Fault) and t_1 three times. Firstly, the covariance matrix of node positive sequence voltages has been calculated.

The covariance value at the 18th diagonal element is 0.284747, which is also the biggest one in the complete covariance matrix based on node positive sequence voltage of IEEE 39-Bus system. Hence, the Bus18 could be preliminarily determined as one of the probable fault sections.

Let's further solve the eigenvalues of this covariance matrix, see Table III.

TABLE III. THE EIGENVALUES OF COVARIANCE MATRIX BASED ON NODE POSITIVE SEQUENCE VOLTAGE OF IEEE 39-BUS SYSTEM

No.	Eigenvalues	Proportion	Cumulative
1	1.60811522	0.9994	0.9994
2	0.00095701	0.0006	1.0000

Finally, the first principal component is obtained, which can be expressed as,

$$\begin{aligned} f_1 = {} & 0.082698x_1 + 0.170957x_2 + 0.266184x_3 \\ & +0.167048x_4 + 0.125634x_5 + 0.118646x_6 \\ & +0.109797x_7 + 0.109891x_8 + 0.060092x_9 \\ & +0.123968x_{10} + 0.117597x_{11} + 0.126606x_{12} \\ & +0.135632x_{13} + 0.153526x_{14} + 0.178631x_{15} \\ & +0.205746x_{16} + 0.308420x_{17} + 0.420796x_{18} \\ & +0.129685x_{19} + 0.102212x_{20} + 0.165739x_{21} \\ & +0.134269x_{22} + 0.132319x_{23} + 0.197292x_{24} \\ & +0.160474x_{25} + 0.198438x_{26} + 0.247830x_{27} \\ & +0.150429x_{28} + 0.132497x_{29} + 0.109699x_{30} \\ & +0.077311x_{31} + 0.079436x_{32} + 0.087002x_{33} \\ & +0.081857x_{34} + 0.101435x_{35} + 0.079830x_{36} \\ & +0.110924x_{37} + 0.099888x_{38} + 0.022021x_{39} \end{aligned} \tag{15}$$

Because the cumulative of the first principal component has reached 99.94%, in this place, we only need to extract the first principal component.

Based on a comprehensive analysis of these present results, one can conclude as follows: From the feature of the first principal component, Bus18 corresponds with variable x_{18}, and the coefficient of x_{18} is 0.420796, which is the biggest. Consequently, Bus18 is just the fault component. This conclusion is also entirely identical with the fault set in advance.

D. The analysis of noise influence

In our researches, the basic idea is to extract the fault features from the coefficients of principal component. Because we have adopted the standard deviation of voltage vector and current vector as 0.006 and the mean error as 0, we will still carry on deep analysis of the noise effect on the feature of coefficients in the similar way. Therefore, the mean level of the coefficients error has been chosen to depict the special effect of the noise.

Based on the node positive sequence voltage and node negative sequence voltage respectively in the standard test example of IEEE9-Bus system, the effect of the noise is calculated and shown in the Fig. 3 and Fig. 4.

Wherein, the solid line and dotted line are used to represent the feature of the principal coefficients in the theoretical and noise situation respectively, and the dashed line is applied to depict the error existing in these two situations. In a conclusion, the mean error levels respectively which are corresponding to the node positive sequence voltage and node negative sequence voltage are 2.6289% and 3.1891% from the principal component features.

In the similar way, the mean error level of the node positive sequence voltage from the IEEE39-Bus system test example is also analyzed, which is 1.6362% shown in the Fig. 5.

These instances have fully proven that the fault identification can be performed successfully by principal component analysis and calculation. Under the influence of noise, the results are still accurate and reliable. So, the principal components fault identification has strong anti-interference ability and great redundancy.

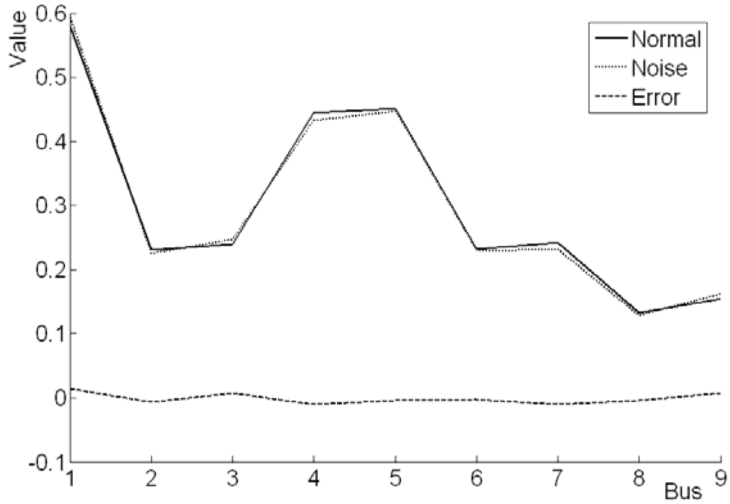

Figure 3. The effect of noise on node positive sequence voltage in IEEE9-Bus system

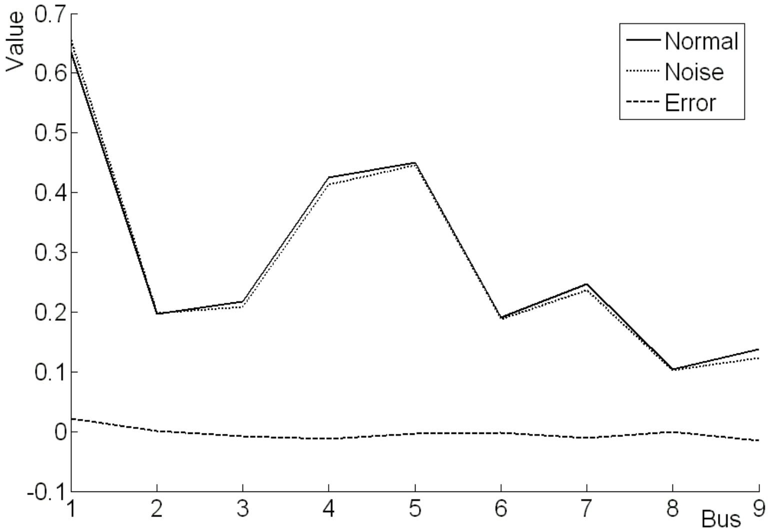

Figure 4. The effect of noise on node negative sequence voltage in IEEE9-Bus system

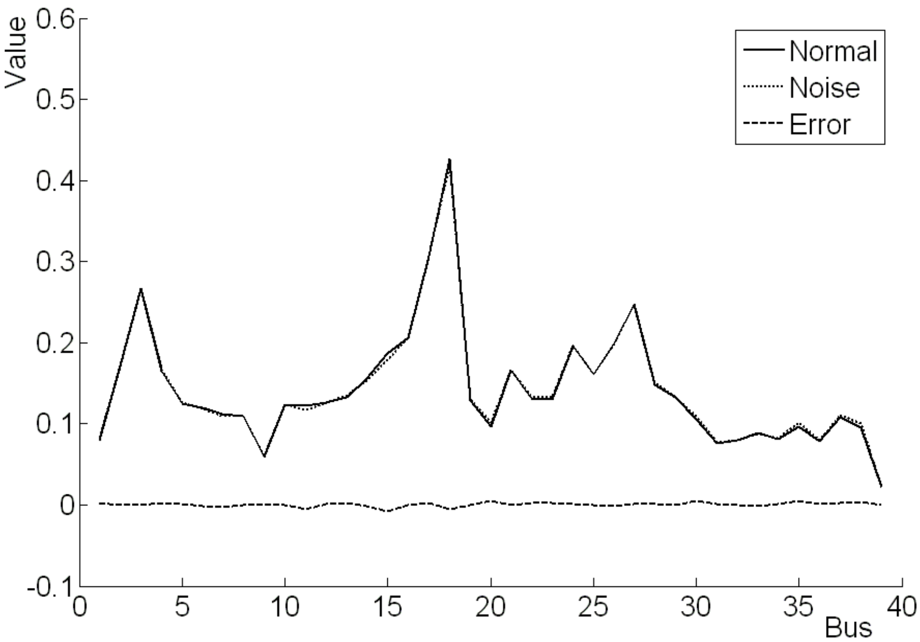

Figure 5. The effect of noise on node positive sequence voltage in IEEE39-Bus system

V. CONCLUSION

Principal component analysis uses a mathematical procedure that transforms a set of correlated response variables into a new set of uncorrelated variables. PCA can be used to reduce the dimensionality of multivariate data. In our researches, the principal components fault identification based on wide area measurements has been discussed carefully.

Electric power system is one of the most complex artificial systems in this world. The increasing complexity of modern power system has brought great difficulty to the coordination of the relay protection, especially for the backup protection. When a disturbance happens in the power system, the voltage and current information in the whole network will change accordingly. As to relay protection, how to extract the fault features and to detect the fault from the large amount of measurement information supplied by the WAMS is the key problem for the backup protection based on the identification of fault. In this paper, the principal components analysis theory is introduced into the field of fault detection to locate precisely the fault by mean of the voltage and current phasor data from the PMUs.

Massive simulation experiments have fully proven that the fault identification can be performed successfully by principal component analysis and calculation. Our researches indicate that the variable with the biggest coefficient in principal component usually corresponds to the fault. Under the influence of noise, the results are still accurate and reliable. So, the principal components fault identification has strong anti-interference ability and great redundancy. Of course, because of the complexity of different types of faults in electric power system, there still exists some problem need intensive study. According to the real-time data from the real test system, the treatment of signal to noise ratio (SNR) with uncertain noise level needs further study in our following work.

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