Inter ISO Market Coordination by Calculating Border Locational Marginal Prices

Aleksa B. BABIĆ[1], Andrija T. SARIĆ[2,3]
[1]*Department for Transmission Planning, South California Edison, 2244 Walnut Grove Ave. GO1, Quad 1D, Rosemead, CA 91770, USA.*
[2]*Faculty of Technical Sciences, University of Kragujevac, Svetog Save 65, 32000 Čačak, Serbia.*
[3]*Faculty of Technical Sciences, University of Novi Sad, Trg Dositeja Obradovića 6, 21000 Novi Sad, Serbia.*
andrija.saric@ftn.kg.ac.rs; *asaric@uns.ac.rs*

***Abstract*—In this paper the methodology for solving Locational Marginal Price (LMP) differences (inconsistency of LMPs) that arise at the boundary buses between separate power markets is proposed. The algorithm developed enables us to obtain consistent LMP values at the boundary buses between interconnected ISOs. A Primal-Dual Interior Point based optimal power flow (OPF) is applied, with complete set of power system physical limit constraints, to solve a regional spot market. The OPF is implemented such that producer and consumer behaviors are modeled simultaneously, while the welfare is maximized. In this paper a generalized methodology for multiple ISOs case is proposed and later it is practically applied on two interconnected independent entities. The algorithm for approximation of cost coefficients of generators and dispatchable loads for neighboring ISOs is proposed. The developed algorithm enables participating ISOs to obtain LMPs at the boundary buses with other interconnected ISOs. By controlling interchange of electric power at the scheduled level, regional spot markets are resolved eliminating possible exercise of market power by individual interconnected ISOs. Results of proposed methodology are tested on the IEEE 118-bus power system.**

***Index Terms*—Inter ISO Market Coordination, Border Locational Marginal Price (LMP), Optimal Power Flow (OPF).**

NOMENCLATURE

Note that vectors and matrices are denoted in bold letters.

Variables

$\boldsymbol{g}$ Active and reactive power balance equations

C Bid cost function for generator and dispatchable load buses

$\boldsymbol{d} = \left[\boldsymbol{\lambda_r}^{\mathrm{T}} \quad \boldsymbol{\lambda}^{\mathrm{T}} \quad \boldsymbol{\mu}_{\boldsymbol{LVC}}^{\mathrm{T}} \quad \boldsymbol{\lambda}_{\boldsymbol{FE}}^{\mathrm{T}} \quad \boldsymbol{\pi}_{\boldsymbol{PQC}}^{\mathrm{T}} \right]^{\mathrm{T}}$ Vector of dual variables

$\boldsymbol{D} = \left[\boldsymbol{P_G}^{\mathrm{T}} \quad \boldsymbol{Q_G}^{\mathrm{T}} \quad \boldsymbol{P_{DL}}^{\mathrm{T}} \right]^{\mathrm{T}}$ Vector of decision variables

f Generalized objective function of the OPF algorithm

$\boldsymbol{h}_{\mathrm{e}}$, $\boldsymbol{h}_{\mathrm{ie}}$, $\boldsymbol{b}$ Sets of equality, inequality and bounds constraints in OPF formulation, respectively

L_n Number of tie-lines from n-th ISO

$\boldsymbol{P_G}$, $\boldsymbol{Q_G}$ Vectors of amount of the active and reactive power produced, respectively

$\boldsymbol{P_{DL}}$, $\boldsymbol{P_L}$ Vectors of amount of the active dispatchable and constant power consumed, respectively

This work was supported in part by the Ministry of Education and Science of the Republic of Serbia within project III-42004.

Digital Object Identifier 10.4316/AECE.2013.02008

P_ℓ Active power flow on individual transmission branch (line or transformer) usually constrained by thermal limit

$\boldsymbol{s} = \left[\boldsymbol{s}_{\mathrm{ie}}^{\mathrm{T}} \quad \boldsymbol{s}_{\mathrm{b}}^{\mathrm{T}} \right]^{\mathrm{T}}$ Vector of slack variables for inequality and decision variable bounds constraints, respectively

$\boldsymbol{S} = \left[\boldsymbol{e}^{\mathrm{T}} \quad \boldsymbol{f}^{\mathrm{T}} \right]^{\mathrm{T}}$ Vector of state variables

$V = e^2 + f^2$ Bus voltage in rectangular form (e and f are voltage rectangular coordinates)

$\boldsymbol{x} = \left[\boldsymbol{S}^{\mathrm{T}} \quad \boldsymbol{d}^{\mathrm{T}} \quad \boldsymbol{D}^{\mathrm{T}} \quad \boldsymbol{s}^{\mathrm{T}} \right]^{\mathrm{T}}$ Vector of state, dual, decision and slack variables, respectively

$\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, $\boldsymbol{\pi}$ Corresponding dual vectors for equality, inequality and bounds constraints, respectively

W Welfare cost function of the OPF algorithm

$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_P \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} LMP_P \\ LMP_Q \end{bmatrix}$ Vector of Locational Marginal Prices for active and reactive powers

$\Im$ Lagrangian function

$\nabla \Im$, $\nabla^2 \Im$ Gradient and Hessian of the Lagrangian function

Indices

FE Set of active power fixed exchange (equality constraints)

i, j, k Current indices for generator, dispatchable (constant) load and all buses, respectively

(k) Iteration count

ℓ Current index for branches

LVC Set of voltage magnitude and active power transmission line flow inequality constraints

n Current index for ISOs

n_c Total number of inequality constraints

Max, Min Maximum and minimum values, respectively

PQC Set of constraints for amount of active and reactive power produced and the amount of active power consumed

P, Q Active and reactive powers, respectively

r Reference bus

Parameters

a, b, c Coefficients of cost curves submitted to the electric power market for producers (generators) and consumers (loads)

G, B Conductance and susceptance elements of bus admittance matrix

lb, **ub** Lower and upper bounds, respectively

n_{bus}, n_{line} Total numbers of buses and lines in interconnection, respectively

n_{ISO} Total numberof ISOs

n_G, n_{DL}, n_L Total numbers of generators, dispatchable and constant loads in interconnection, respectively

Abbreviations

AC	Nonlinear power flow
DC	Linear power flow
ILMPCM	Inter ISO LMP Coordination Method
ISO	Independent System Operator
TSO	Transmission System Operator
LMP	Locational Marginal Price
Real, Imag	Real and imaginary parts of complex variable, respectively
OPF	Optimal Power Flow

I. INTRODUCTION

The Locational Marginal Prices (LMPs) inconsistency occurs at the border of interconnected entities operated by different system operators, such as Independent System Operators (ISOs) in US practice, or Transmission System Operators (TSOs) in European practice. Given two or more interconnected entities, loads in one territory may wish to purchase cheaper electricity from production capacities in a neighboring territory. Loop-flows, inter zonal congestion and attendant losses occur as a consequence. Due to limited amount of information, as imposed by a deregulated environment, inconsistent LMPs arise and it prevents two coordinated entities in achieving a common uniform solution. This problem is known as "*seams problems*" (in US terminology) [1-4], or "*cross-border congestion*" (in European terminology) [5]. Seams problems have been in existence since before and after deregulation.

The problem of inconsistent LMPs that arises in deregulated electric utility markets addresses following issues [1-5]:

- Inconsistent power market design between the interconnected ISOs.
- Problem of accumulating electric power transfer charges on the territory of several ISOs.
- Obligation to receive a Transmission Congestion Right by each of the ISOs through which territory a scheduled transaction is anticipated to be transferred.
- Reduction in the inter ISO transfer capacity due to line outages.
- Slow LMP convergence when several interconnected ISOs try to achieve a coordinated optimal solution for the entire grid.

Main problem arises in achieving the LMP convergence between the ISOs to provide a common optimal solution for the entire power system to cope with seam problem between interconnected ISOs [4]. A major issue is that an ISO is reluctant to release all of its power system data in order to avoid the exercise of market power by competing power producers.

As far as previous work on problem of market practices at the border of several ISOs is concerned, a DC approach is used by the Cadwalader et al [5]. It assumes that an ISO enforces transmission constraints on its' own territory and the effects of transmission congestion of the neighboring ISOs are incorporated in the objective function. In addition, each ISO has to approximate bid curve coefficients of the neighboring ISOs involved in inter regional congestion relief process [5]. Approximation is based upon the data exchange over the central platform. In [6,7] an approach is proposed in which a power system is decomposed into overlapping regions and in each individual region Optimal Power Flow (OPF) is solved by enforcing equality and inequality constraints for its' own region and intersecting regions (tie-lines between two interconnected power systems). In [8,9] an approach is proposed in which fictitious nodes are introduced at the border of two neighboring power systems and in each individual region linear (DC) based OPF is solved by enforcing equality and inequality constraints for its' own region and constraints on fictitious nodes.

The performance of the different inter-regional interchange systems in USA, the alternative market procedures that could improve this performance, and preliminary economic benefit estimates from these improvements are explained in details in [10,11]. This problem should be also very important in European regions (such as, for example, central Europe, Scandinavia or Balkan peninsula), then the future (inter-)regional electricity markets will be fully established and coordinated [12-14].

Therefore, in this paper is assumed that all allowed information has to be exchanged between participating ISOs and OPF is resolved until all bus LMP differences in two successive outer iterations fall within an acceptable tolerance. In each step OPF is solved for all of the participating ISOs sequentially, and information between OPF solutions (inner algorithmic step) and the outer algorithmic step (LMP convergence criteria) is exchanged such that approximations of generator and dispatchable load prices on the territory of the neighboring ISOs are calculated. In this paper, the assumption is followed that only the generator and variable load cost coefficients of the competing neighbors are not available to the rest of the ISOs, since a power system network configuration data is available within an ISO's interconnection. A generalized LMP based decomposition algorithm is proposed and applied on two ISOs test case. LMP market results are obtained using the full nonlinear (AC) OPF based algorithm [15-18].

Finally, the proposed problem solution can be characterized as follows:

1) In general, solving the regional OPF requires a large number of iteration steps to achieve LMP convergence at the bordering buses for the uniform solution of the electric power grid controlled by two or more ISOs. The objective is to achieve a uniform LMP convergence and to match the solutions of the two ISOs with a solution of the system treated as a joint entity for the same particular power system.
2) As a consequence of scheduling a power interchange between the several ISOs and enforcing a scheduled net active power flow over the tie-lines, the amount of loop-flow is reduced as a consequence of power transfer schedule control and exercise of market power by participating ISOs is avoided.

The paper is organized as follows: The new methodology for solving problem of inconsistent LMPs at the border of neighboring ISOs based on the full nonlinear (AC) based OPF is formulated in *Section II*. LMP decomposition with interpretation of the duals of the power balance equality constraints, physical inequality (branch power flows and bus voltages) constraints and power schedule interchange equality constraints is shown in *Section III*. Approximation of the cost coefficients for neighboring ISOs is given in *Section IV*. Numerical results are shown in *Section V*, while concluding remarks are provided in *Section VI*. In two appendices (*A* and *B*), the basic equation for applied OPF formulation and their Primal-Dual Interior Point based optimization algorithm, respectively, used in this paper are explained in more detail.

II. Methodology

A. Methodology Specifics

In the proposed methodology each of the ISOs has all of the data available about the power system, except generator and load cost curves of the neighboring ISOs [4]. After an OPF is solved by the regional ISOs through iteration steps until reaching the LMP convergence, power transfer between the neighboring ISOs is going to be scheduled based upon the agreement between the two ISOs and it could be in either direction. Then, each ISO is going to solve an OPF again until a coordinated solution is achieved. The specifics of this new methodology are following:

1) The information not shared between ISOs is generator and dispatchable load cost curves.
2) Each ISO solves an OPF for the entire interconnected power system, but it has to rely on approximating generator and dispatchable load cost coefficients of the other ISOs, as it is shown in *Section IV*.
3) The local optimal solution is reached, due to non convexity of the Primal-Dual Interior Point based OPF and therefore the local market equilibrium is achieved.
4) Due to scheduled electric power transfer, inter ISO loop flows are controlled.
5) OPF solutions by the neighboring ISOs match each other completely, and match as well with the OPF solution for the entire grid run from a standpoint of a joint entity.

B. Inter ISO LMP Coordination Method

The methodology developed in this paper is called Inter ISO LMP Coordination Method (ILMPCM). On the **Central Platform**, displayed in Fig. 1, LMPs, generation active and reactive powers ($\boldsymbol{P_G}$ and $\boldsymbol{Q_G}$) and dispatchable load active powers ($\boldsymbol{P_{DL}}$) are shared by regional ISOs. Each ISO sends just mentioned data, obtained from its full nonlinear (AC) OPF solution for the entire interconnection, to the **Central Platform**. Such information can be shared by the neighboring ISOs. The ILMPCM algorithm is presented in more details in Figs. 1,2.

This new algorithm runs as follows:

1) Initial set of LMPs, generator active/reactive power data and dispatchable load active power data are loaded for the entire interconnection (for all considered ISOs).
2) Each ISO solves its own OPF, with scheduled transfers of active power, for the entire interconnection, by using approximated generator and dispatchable load cost coefficients in the neighboring territories (ISOs). Note that due to this approximation, the OPF may have a slower convergence during the first couple of coordination steps (see presented results in *Section V*).
3) Each one of the interconnected ISOs sends its OPF solution, bus LMPs, generator active/reactive power data and dispatchable load active power data to be displayed on the **Central Platform** (see Fig. 1).
4) Check for the convergence of LMPs:

   In successive outer iterations, check the absolute value of LMP difference on all buses of interconnected ISOs (on both sides of the tie-lines). This value on any bus should be below specified criteria for the converged case (ε in Fig. 2).

   If LMP differences converge, go to Step 5, else if LMP differences do not converge, go back to Step 2.
5) Terminate algorithm.

Central Platform could be a computer hub supervised by an authorized representative of participating ISOs. Pieces of information posted on the **Central Platform** typically shared by ISOs are the following ones:

1.) LMPs of all buses obtained from solved OPF for interconnected ISOs.
2.) Generator's active power outputs obtained from the OPF solution for each of the ISOs.
3.) Dispatchable load's active power outputs obtained from the OPF solution for each of the ISOs.

Note that keeping the interchange schedule between ISOs does not allow transfer of market power from one ISO to another.

Finally, on a diagram in Fig. 2 is shown the overall iteration process of the proposed algorithm.

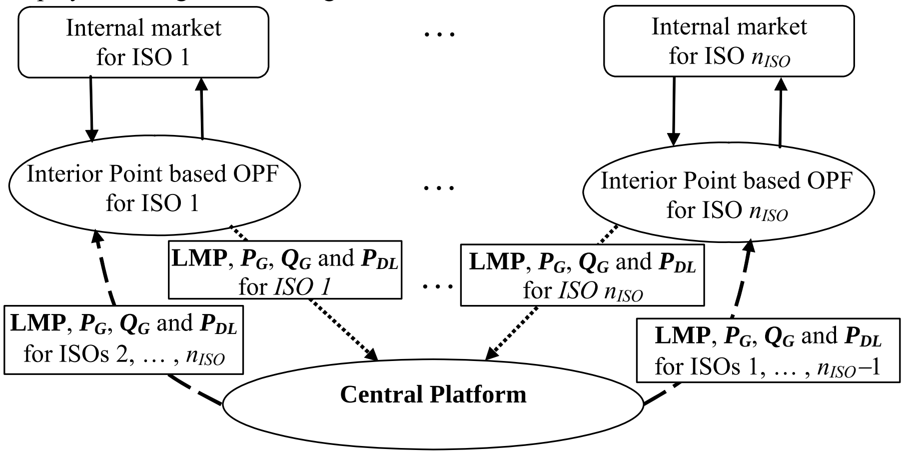

Figure 1. Inter ISO LMP Coordination

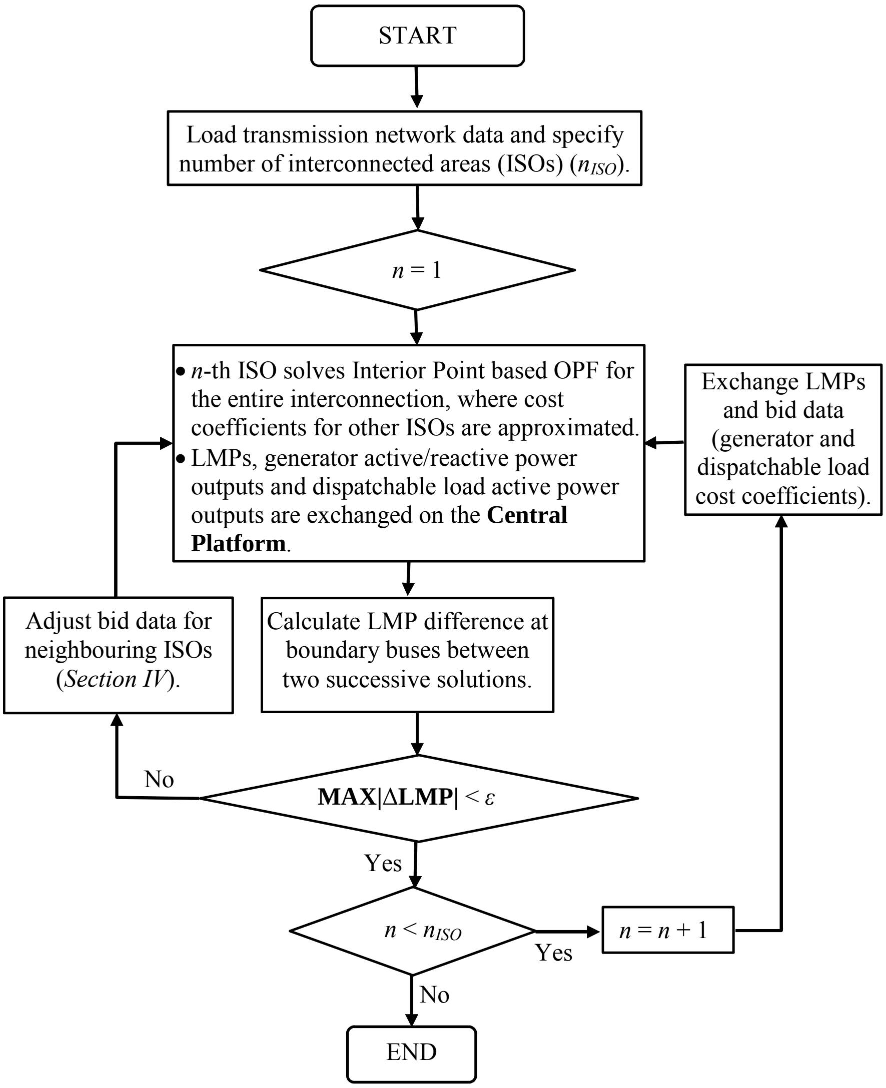

Figure 2. Flow-chart of proposed inter ISO formulation and solution for problem of inconsistent LMPs at boundary buses

Based on derived power flow equations in the *Appendix A*, the OPF is formulated as follows:

$$\underset{\boldsymbol{P_G},\,\boldsymbol{Q_G},\,\boldsymbol{P_{DL}}}{\text{Minimize}}: \quad W(\boldsymbol{P_G},\ \boldsymbol{P_{DL}}) \qquad \text{(A1c)}\rightarrow\text{(1)}$$

subject to:

1. $\boldsymbol{g_r}(e_r,\ f_r,\ \boldsymbol{e},\ \boldsymbol{f}) = \boldsymbol{0}: \quad \leftrightarrow \boldsymbol{\lambda_r} \qquad \text{(A3)}\rightarrow\ (2)$
2. $\boldsymbol{g}(P_{Gr},\ Q_{Gr},\ \boldsymbol{P_G},\ \boldsymbol{Q_G},\ \boldsymbol{P_{DL}},\ e_r,\ f_r,\ \boldsymbol{e},\ \boldsymbol{f}) = \boldsymbol{0}:$
$\leftrightarrow \boldsymbol{\lambda} \qquad \text{(A2)}\rightarrow\ (3)$
3. $\boldsymbol{LVC}(e_r,\ f_r,\ \boldsymbol{e},\ \boldsymbol{f}) \le \boldsymbol{0}: \quad \leftrightarrow \boldsymbol{\mu_{LVC}} \qquad \text{(A4,5)}\rightarrow\ (4)$
4. $\boldsymbol{FE}(e_r,\ f_r,\ \boldsymbol{e},\ \boldsymbol{f}) = \boldsymbol{0}: \quad \leftrightarrow \boldsymbol{\lambda_{FE}} \qquad \text{(A6)}\rightarrow\ (5)$
5. $\begin{array}{l}\boldsymbol{PQC}(P_{Gr},\ Q_{Gr},\ \boldsymbol{P_G},\ \boldsymbol{Q_G},\ \boldsymbol{P_{DL}}) - \mathbf{ub} \le \boldsymbol{0} \\ \mathbf{lb} - \boldsymbol{PQC}(P_{Gr},\ Q_{Gr},\ \boldsymbol{P_G},\ \boldsymbol{Q_G},\ \boldsymbol{P_{DL}}) \le \boldsymbol{0}\end{array}:$

$$\leftrightarrow \boldsymbol{\pi_{PQC}} = \begin{bmatrix} \boldsymbol{\pi_{PQC}^{\mathbf{lb}}} \\ \boldsymbol{\pi_{PQC}^{\mathbf{ub}}} \end{bmatrix} \qquad \text{(A7)}\rightarrow\ (6)$$

OPF formulation for the ISOs $n = 2, 3, \cdots, n_{ISO}$ is identical to that of the ISO 1. The only difference lies in the approximated coefficients of the generator and dispatchable load cost curves.

According to Fig. 1, system of equations below is solved by each of the n_{ISO} ISOs with exchange of already mentioned information in between each OPF solution (the solution algorithm in more details is described in *Section IV*):

$$\Delta\boldsymbol{x} = -[\nabla^2\Im]^{-1}\nabla\Im\,; \qquad (7)$$

$$\boldsymbol{x}^{(k+1)} = \boldsymbol{x}^{(k)} + \Delta\boldsymbol{x}\,, \qquad (8)$$

where Gradient (∇) and Hessian (∇^2) of Lagrangian function ($\Im$) for optimization problem (1)-(6) are defined in *Appendix B*.

Iterative calculation using (7) and (8) is executed until the maximum absolute value of $\Delta\boldsymbol{x}$ falls below pre-specified tolerance criteria (typically equal to 10^{-3}).

It is important to note that from converged OPF solution the LMPs for active and reactive power at generator and dispatchable load buses should be obtained directly as $\boldsymbol{LMP} = \boldsymbol{\lambda}$.

Once the OPF is solved by a particular ISO, next ISO proceed to solve the OPF utilizing the shared information through the **Central Platform**. This ISO approximates the generator and dispatchable load quadratic cost coefficients on territory of the neighboring ISOs by the methodology proposed in *Section IV*.

III. LMP Decomposition and Interpretation of Duals of Active Power Interchange Schedules

This section reviews the concept of LMPs and it shows that by applying methods from the linear algebra it is possible to decompose LMPs into three components.

It is clearly seen in the formulated OPF in (1)-(6), that there is a separate group of primal variables pertaining to the reference bus (r). Set of equality constraints ($\boldsymbol{g_r} = \mathbf{0}$ in (2)) represent the active and reactive power balance equations for the reference bus. It stems from the separate group of primal variables just mentioned.

Once the first order necessary Karush-Kuhn-Tucker optimality condition for Lagrangian in (B5) are met ($\partial\Im/\partial\boldsymbol{S} = \mathbf{0}$), it can be shown, for example, that $\boldsymbol{LMP_P} = \boldsymbol{\lambda_P}$ can be represented as a marginal welfare cost of each additional amount of active power produced by generators ($\boldsymbol{P_G}$), or consumed by dispatchable loads ($\boldsymbol{P_{DL}}$) at any bus in the power system with equality ($\boldsymbol{g_r}$ and $\boldsymbol{g}$), inequality ($\boldsymbol{LVC}$ and $\boldsymbol{FE}$) and lower/upper bounds ($\boldsymbol{PQC}$) constraints satisfied:

Generator buses:

$$\frac{\partial[C(P_{Gi}) + \boldsymbol{g_r}^{\mathrm{T}}\boldsymbol{\lambda_r} + \boldsymbol{g}^{\mathrm{T}}\boldsymbol{\lambda} + \boldsymbol{LVC}^{\mathrm{T}}\boldsymbol{\mu_{LVC}} + \boldsymbol{FE}^{\mathrm{T}}\boldsymbol{\lambda_{FE}} + \boldsymbol{PQC}^{\mathrm{T}}\boldsymbol{\pi_{PQC}}]}{\partial P_{Gi}} = 0;$$

$$i = 1, 2, \cdots, n_G; \tag{9a}$$

Dispatchable load buses:

$$\frac{\partial[C(P_{DLj}) + \boldsymbol{g_r}^{\mathrm{T}}\boldsymbol{\lambda_r} + \boldsymbol{g}^{\mathrm{T}}\boldsymbol{\lambda} + \boldsymbol{LVC}^{\mathrm{T}}\boldsymbol{\mu_{LVC}} + \boldsymbol{FE}^{\mathrm{T}}\boldsymbol{\lambda_{FE}} + \boldsymbol{PQC}^{\mathrm{T}}\boldsymbol{\pi_{PQC}}]}{\partial P_{DLj}} = 0;$$

$$j = 1, 2, \cdots, n_{DL}. \tag{9b}$$

By following the same line of reasoning, the inequality constraint multipliers ($\boldsymbol{\mu_{LVC}}$) and bound constraint multipliers ($\boldsymbol{\pi_{PQC}}$) represent a marginal welfare cost of the system with respect to corresponding operational limits.

Under the OPF formulation applied in this paper, from $\partial\Im/\partial\boldsymbol{S} = \mathbf{0}$ the LMPs are decomposed into three components. There exists a component due to generation/load (including the losses), transmission congestion and bus voltage constraints, and there is an additional component due to power interchange between ISOs schedule [19]:

$$\begin{bmatrix} \nabla_{\boldsymbol{e}}(\boldsymbol{g_r})^{\mathrm{T}} \\ \nabla_{\boldsymbol{f}}(\boldsymbol{g_r})^{\mathrm{T}} \end{bmatrix}\boldsymbol{\lambda_r} + \begin{bmatrix} \nabla_{\boldsymbol{e}}(\boldsymbol{g})^{\mathrm{T}} \\ \nabla_{\boldsymbol{f}}(\boldsymbol{g})^{\mathrm{T}} \end{bmatrix}\boldsymbol{\lambda} + \begin{bmatrix} \nabla_{\boldsymbol{e}}(\boldsymbol{LVC})^{\mathrm{T}} \\ \nabla_{\boldsymbol{f}}(\boldsymbol{LVC})^{\mathrm{T}} \end{bmatrix}\boldsymbol{\mu_{LVC}}$$

$$+\begin{bmatrix} \nabla_{\boldsymbol{e}}(\boldsymbol{FE})^{\mathrm{T}} \\ \nabla_{\boldsymbol{f}}(\boldsymbol{FE})^{\mathrm{T}} \end{bmatrix}\boldsymbol{\lambda_{FE}} = \mathbf{0} \begin{matrix} :\boldsymbol{e} \\ :\boldsymbol{f} \end{matrix}. \tag{10}$$

The system of equations (10) can be solved for the LMPs by summing the three already mentioned components:

$$\boldsymbol{\lambda} = -\begin{bmatrix} [\nabla_{\boldsymbol{e}}(\boldsymbol{g})]^{\mathrm{T}} \\ [\nabla_{\boldsymbol{f}}(\boldsymbol{g})]^{\mathrm{T}} \end{bmatrix}^{-1}$$

$$\left(\begin{bmatrix} [\nabla_{\boldsymbol{e}}(\boldsymbol{g_r})]^{\mathrm{T}} \\ [\nabla_{\boldsymbol{f}}(\boldsymbol{g_r})]^{\mathrm{T}} \end{bmatrix}\boldsymbol{\lambda_r} + \begin{bmatrix} [\nabla_{\boldsymbol{e}}(\boldsymbol{LVC})]^{\mathrm{T}} \\ [\nabla_{\boldsymbol{f}}(\boldsymbol{LVC})]^{\mathrm{T}} \end{bmatrix}\boldsymbol{\mu_{LVC}} + \begin{bmatrix} [\nabla_{\boldsymbol{e}}(\boldsymbol{FE})]^{\mathrm{T}} \\ [\nabla_{\boldsymbol{f}}(\boldsymbol{FE})]^{\mathrm{T}} \end{bmatrix}\boldsymbol{\lambda_{FE}}\right). \tag{11}$$

The inverted matrix in (11) is reminiscent of the inverse Jacobian matrix from the Newton-Raphson power flow [15]. By definition, a marginal reference bus (*r*) has prices $\boldsymbol{\lambda_r}$ for active and reactive power only due to generation/load (including the losses) component, while the transmission congestion/bus voltage constraints and the fixed exchange over tie-lines components are both equal to zero.

Based on above derivation, it is possible to distinguish (generation/load+loss) components, transmission congestion/bus voltage components and fixed exchange components at all other buses in the power system:

$$\boldsymbol{LMP} = \boldsymbol{LMP}_{\mathrm{Gen/Load+Loss}} + \boldsymbol{LMP}_{\mathrm{Cong/Volt}} + \boldsymbol{LMP}_{\mathrm{FE}}. \tag{12}$$

In the case of no transmission congestion and violated bus voltage constraints, the bus LMPs consist of (generation/load+loss) components and the fixed exchange components:

$$\boldsymbol{LMP} = \boldsymbol{LMP}_{\mathrm{Gen/Load+Loss}} + \boldsymbol{LMP}_{\mathrm{FE}}. \tag{13}$$

Therefore, the fixed exchange component can be interpreted as difference between the LMP and (generation/load+loss) component at each bus:

$$\boldsymbol{LMP}_{\mathrm{FE}} = \boldsymbol{LMP} - \boldsymbol{LMP}_{\mathrm{Gen/Load+Loss}}. \tag{14}$$

This is in accordance with observations made in Table III that the dual of the fixed exchange equals the LMP difference at the line ends unless congestion occurs at any of the tie-lines. Based on the sign of the value of the dual of the active power fixed exchange equality constraint economic stimulus of interchange could be assessed, but this is not a subject to be discussed further in this paper.

IV. Cost Coefficients Approximation for Neighboring ISOs

Given that a generator or load bid bus does not belong to the ISO control area where its' operator solves the OPF for the entire interconnection, cost coefficients on those buses have to be approximated using data available on the **Central Platform** (Fig. 1), provided as a part of the OPF solution by the neighboring ISOs.

According with Lagrangian defined by (B5) and Lagrange multiplier theory, bus LMP is going to be set equal to sum of corresponding first order derivative of the bid cost function at point of the optimal active power (for neighboring ISOs obtained from the **Central Platform**) and duals of active power bounds constraints (π_{PC}), or:

$$LMP_{Pi} = \left.\frac{\partial C(P_{Gi})}{\partial P_{Gi}}\right|_{P_{Gi}=P_{Gi}^*} + \pi_{PCi}^{ub,*} - \pi_{PCi}^{lb,*}$$

$$= b_{Gi} + 2c_{Gi}P_{Gi}^* + \pi_{PCi}^{ub,*} - \pi_{PCi}^{lb,*};\ \ i = 1, 2, \cdots, n_G; \tag{15a}$$

$$LMP_{Pj} = \left.\frac{\partial C(P_{DLj})}{\partial P_{DLj}}\right|_{P_{DLj}=P_{DLj}^*} + \pi_{PCj}^{ub,*} - \pi_{PCj}^{lb,*}$$

$$= b_{DLj} - 2c_{DLj}P_{DLj}^* + \pi_{PCj}^{ub,*} - \pi_{PCj}^{lb,*};\ \ j = 1, 2, \cdots, n_{DL}, \tag{15b}$$

where $P_{Gi} = P_{Gi}^*$ ($P_{DLj} = P_{DLj}^*$) and π_{PCi}^* (π_{PCj}^*) denotes optimal generation (dispatchable load) active powers and corresponding lower/upper bound duals for Primal-Dual Interior Point based OPF solution.

Assuming the typical cost coefficient values $c_{Gi} = c_{DLj} = 0.1$ [15], we have:

$$b_{Gi} = LMP_{Pi} - \pi_{PCi}^{ub,*} + \pi_{PCi}^{lb,*} + 0.2P_{Gi}^*;\ i = 1, 2, \cdots, n_G; \tag{16a}$$

$$b_{DLj} = LMP_{Pj} - \pi_{PCj}^{ub,*} + \pi_{PCj}^{ub,*} - 0.2P_{DLj}^*;\ j = 1, 2, \cdots, n_{DL}. \tag{16b}$$

It has been experimentally proved in numerous simulations on the analyzed 118-bus power system that such an approximation leads to the LMP convergence, but still rigorous mathematical proof of it does not exist [19].

V. NUMERICAL STUDIES

The IEEE 118-bus [20] power system is used for validity testing of the proposed algorithm for dealing with inconsistent LMPs at ISO's boundary buses.

The entire algorithm is developed using MATLAB 7.0.

The main objective is to control the interchange of active power flows between the interconnected entities to obtain consistent LMPs between ISOs by using the full nonlinear (AC) OPF. In this way the exercise of market power by two ISOs is avoided.

The test system that is analyzed is divided into two ISOs. Interconnection is complex and there are ten tie-lines connecting the two ISOs (see Fig. 3). Total number of bidders is sixty four. Fifty four bidders come from power plants and ten bidders come from dispatchable loads. Rests of the loads are price inelastic. Summary of basic data for two interconnected ISOs are presented in Table I.

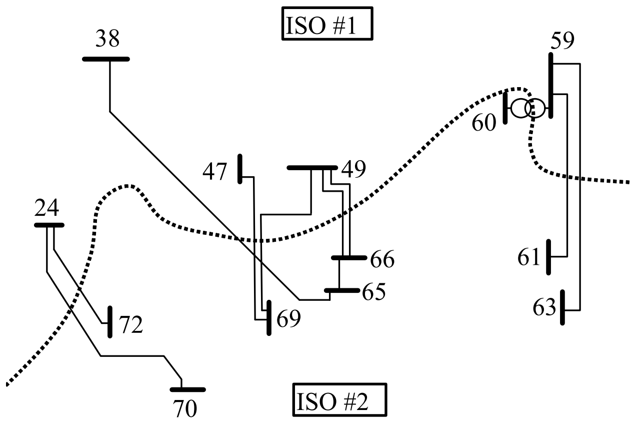

Figure 3. Part of single-line diagram of the IEEE 118-bus test system (only with shown tie-lines and boundary buses between ISO #1 and ISO #2)

In Table II is provided a summary of dispatch data of coordinated solution between the two ISOs.

In order to resolve the problem of LMP inconsistencies it is necessary to control the net active power flow over the tie-lines, when each of the two ISOs solves the OPF. In the numerical experiment, the total net interchange of active power is set to 0 MW (see the last row in Table III).

Table III gives a summary of the tie-line active power flows and LMPs at both ends of ten tie-lines. If we look at the Table III, the LMP difference between the From-bus and To-bus ends equals the absolute value of the dual of the active fixed power exchange – constraint unless congestion occurs as it is in the case of a transmission line connecting buses 24 and 70. This is in accordance with equation (11). The dual of total active power interchange equality constraint is equal to −7.002 $/MWh and this same dual of interchange represents an equality constraint multiplier of the sum of branch MW power flows that connect two territories. Difference in LMPs at the ending buses of any transmission line that is part of interchange control equality constraint is equal to interchange equality constraint dual unless it is a congested transmission line. In the case of a congested tie-line that is part of a MW interchange control set of tie-lines, difference in LMPs on that particular tie-line end buses is not equal to value of the dual of interchange equality constraint (5).

To recapitulate how the algorithm proposed in this paper works, ISO #1 solves the OPF for the entire interconnection with approximated cost coefficients on the territory of ISO #2. It sends data for LMPs, generator output active and reactive powers and dispatchable load output active powers to the **Central Platform**. Then, information sent to the **Central Platform** is utilized by the ISO #2 to approximate cost coefficients on the territory of the ISO #1. At the same time ISO #2 uses its' own correct coefficients. Once the approximation is done, the ISO #2 solves OPF for the entire interconnection. At this point the absolute difference between the LMPs for each individual bus at tie-line ends obtained in two successive OPF solutions in the outer algorithmic loop for both ISOs is calculated (see Fig. 2). If the maximum absolute difference is above the specified criteria it is proceeded with another step, in which the ISO #1 is going to utilize information from the **Central Platform** to approximate cost coefficients on the territory of ISO #2 (see *Section IV*) and solve the OPF for the entire interconnection and so on with ISO #2.

In Figs. 4,5 are provided LMP convergence summaries (i.e. absolute differences in LMPs) at the From-bus and the To-bus ends of the tie-lines for the twelve solution steps, respectively, where in each solution step OPF is solved for both ISO #1 and ISO #2. In the OPF solution ISO #1 uses its' own data and approximated data on territory of ISO #2 and vice versa. The reason why such a big absolute difference in LMP value occurs at the buses on the territory of ISO #2 (Fig. 5) is due to the fact that approximations of generators and dispatchable loads cost coefficients are calculated first in the ISO #2 area of the interconnected power system. Such approximations in the first attempt to solve the OPF have an impact on gradient of objective function and at the same time on value of the LMPs.

TABLE I. SUMMARY OF COORDINATED OPF SOLUTION FOR THE TWO ISOS

System Summary for ISO #1 and ISO #2				
Components	Total number of components	Power system component outputs	P [MW]	Q [Mvar]
Buses	118	Total generation capacity	9966.00	11824.00
Generators	54	Generation (actual)	5463.48	560.89
Loads	100	Load	5374.30	1649.00
Branches	186			
Transformers	9	Branch Charging (inj)	–	1345.93
Areas	2	Shunt (inj)	0	152.60

TABLE II. DISPATCH INFORMATION FOR ISO #1 AND ISO #2

Dispatch data for ISO #1		
Total generation capacity	4102 MW	4894 Mvar
Generation (actual)	2741.41 MW	–116.63 Mvar
Load	2712.28 MW	836.00 Mvar
Dispatch data for ISO #2		
Total generation capacity	5864 MW	6930 Mvar
Generation (actual)	2722.07 MW	677.52 Mvar
Load	2662.01 MW	813.00 Mvar

TABLE III. LMPS AT TIE-LINE BOUNDARY BUSES

From-bus ends	To-bus ends	Max active power flow [MW]	Active power flow [MW]	LMP at From-bus ends [$/MWh]	LMP at To-bus ends [$/MWh]	Abs. LMP difference [$/MWh]
24	70	200	200.00	22.74	22.53	0.21
24	72	200	–62.94	22.74	15.79	6.95
38	65	180	180.00	29.66	24.24	5.42
47	69	200	–48.63	31.11	24.11	7.00
49	66	500	–48.27	31.11	24.11	7.00
49	66	500	–48.27	31.11	24.11	7.00
49	69	200	–57.99	31.11	24.11	7.00
59	60	200	42.81	31.30	24.31	6.99
59	61	200	–116.34	31.30	24.30	7.00
59	63	200	–40.37	31.30	24.30	7.00
			0.00			

Start of the iterative process is always critical due to very rough approximation of the cost coefficients for the neighboring ISO. If we assume that ISO #1 starts with correct cost coefficients and we get rough estimates for the ISO #2 bordering LMPs, convergence of the bordering LMP bus differences is impacted by inter ISO interchange control. In the first several iterations we have big LMP differences. Process of interchange control has a significant impact on defined bordering point LMP convergence, where convergence is defined as a difference in LMPs between two successive outer iterations on any bus.

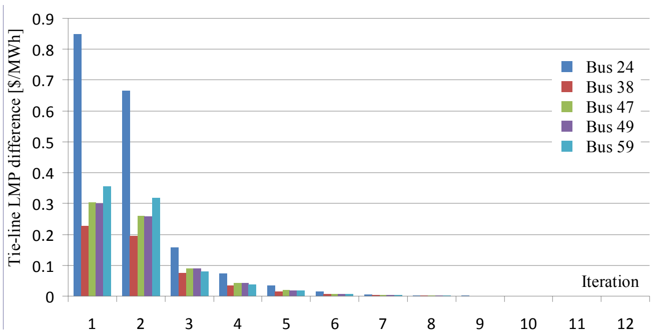

Figure 4. Tie-line LMP difference convergence summary at From-bus ends for ISO #1

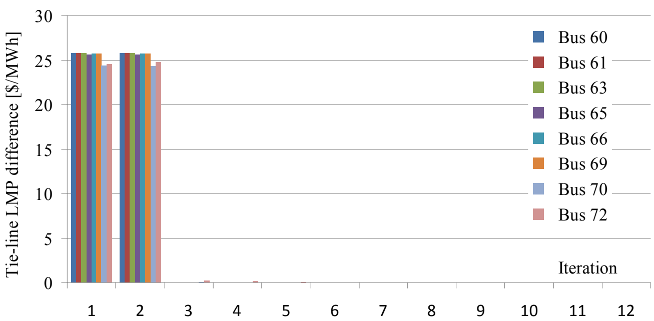

Figure 5. Tie-line LMP difference convergence summary at To-bus ends for ISO #2

VI. CONCLUSION

This paper presents a new method for solving the LMP inconsistencies at boundary buses, where each participating ISO maximizes its welfare on the electricity spot market. The border LMP inconsistency problem is solved by controlling the transfer of active power between the neighboring ISOs. The mathematical method proposed in this paper is an attempt to improve the operation efficiency of power markets. Scheduled exchange of active power between the market participants has been proved to be a better way to solve more efficiently the OPF for the entire interconnection when it has to be coordinated with neighboring ISOs.

This algorithm has been proven to be useful in solving LMP inconsistency by keeping a scheduled interchange of active power between interconnected territories and bringing consistent values of LMPs at the boundary buses.

The proposed algorithm could be used in a day-ahead market schedule to verify contracted schedules for twenty four hours operation planning horizon, satisfying at the same time optimum operation requirements for all interconnected ISOs.

APPENDIX A: DETAILED OPF FORMULATION

1. Bid functions for generator and dispatchable load buses respectively are:

$$C(P_{Gi}) = a_{Gi} + b_{Gi}P_{Gi} + c_{Gi}P_{Gi}^2,\ i = 1, 2, \cdots, n_G; \quad \text{(A1a)}$$

$$C(P_{DLj}) = a_{DLj} + b_{DLj}P_{DLj} - c_{DLj}P_{DLj}^2,\ j = 1, 2, \cdots, n_{DL}, \quad \text{(A1b)}$$

determining the optimization criterion (welfare function in (1)) as:

$$W(\mathbf{D}) = \sum_{j=1}^{n_{DL}} C(P_{DLj}) - \sum_{i=1}^{n_G} C(P_{Gi}). \quad \text{(A1c)}$$

2. Bus (not including the reference bus) active and reactive power balances (equality constraints), respectively are:

$$P_{Gi} - P_{(D)Li} = \sum_{j=1}^{n_{bus}} \left[G_{ij}(e_i e_j + f_i f_j) + B_{ij}(e_i f_j - e_j f_i) \right],$$

$$i = 1, 2, \cdots, n_{bus} \neq r; \quad \text{(A2a)}$$

$$Q_{Gi} - Q_{(D)Li} = \sum_{j=1}^{n_{bus}} \left[G_{ij}(e_j f_i - e_i f_j) - B_{ij}(e_i e_j - f_i f_j) \right],$$

$$i = 1, 2, \cdots, n_{bus} \neq r; \quad \text{(A2b)}$$

3. Reference bus real and reactive power balances (equality constraints), respectively are:

$$P_{Gr} = \sum_{j=1}^{n_{bus}} \left[G_{rj}(e_r e_j + f_r f_j) + B_{rj}(e_r f_j - e_j f_r) \right]; \quad \text{(A3a)}$$

$$Q_{Gr} = \sum_{j=1}^{n_{bus}} \left[G_{rj}(e_j f_r - e_r f_j) - B_{rj}(e_r e_j - f_r f_j) \right]. \quad \text{(A3b)}$$

where voltage components in the reference bus (e_r and f_r) are the constant (not included in the state vector (component $\boldsymbol{S}$)).

Note that active and reactive power system losses are allocated and scheduled to by OPF optimized transactions.

4. Branch active power flow constraints are:

$$\left| P_\ell \right| \le P_\ell^{Max},\ \ell = 1, 2, \cdots, n_{line}. \quad \text{(A4)}$$

5. Bus voltage inequality constraints are:

$$V_k^{Min} \le V_k = \sqrt{e_k^2 + f_k^2} \le V_k^{Max},\ k = 1, 2, \cdots, n_{bus}, \quad \text{(A5)}$$

6. Active power fixed exchange equality constraints are:

$$\sum_{\ell=1}^{L_n} P_\ell = P_n^{sp},\quad n = 1, 2, \cdots, n_{ISO}, \quad \text{(A6)}$$

where P_n^{sp} is specified active power exchange for n-th ISO.

7. Lower/upper limits for decision variables are:

$$P_{Gi}^{Min} \le P_{Gi} \le P_{Gi}^{Max}, \quad i = 1, 2, \cdots, n_G; \tag{A7a}$$

$$Q_{Gi}^{Min} \le Q_{Gi} \le Q_{Gi}^{Max}, \quad i = 1, 2, \cdots, n_G; \tag{A7b}$$

$$P_{DLj}^{Min} \le P_{DLj} \le P_{DL}^{Max}, \quad j = 1, 2, \cdots, n_{DL}; \tag{A7c}$$

$$P_{Lj} = P_{Lj}^{sp}, \quad j = 1, 2, \cdots, n_L, \tag{A7d}$$

where P_{Lj}^{sp} is specified (constant) active power load in j-th bus.

APPENDIX B: PRIMAL-DUAL INTERIOR POINT BASED OPF

The OPF minimizes a specified objective function and at the same time meets physical power system constraints and system limits expressed as equalities, inequalities or both. This paper uses a Primal-Dual Interior Point based nonlinear programming Newton's method with voltage state variables formulated in rectangular coordinates for solving the OPF [15]. This particular kind of OPF is developed as such to maximize the welfare with an additional feature to maintain fixed schedule of the active power exchange over the tie-lines. The welfare maximizing OPF algorithm is useful in power market studies where generators and variable loads are treated as market participants submitting their bids for an auction.

The Primal-Dual Interior Point based OPF is formulated as follows [15]:

$$\underset{\boldsymbol{D}}{\text{Minimize}}: \quad f_\mu = f(\boldsymbol{D}) - \mu \sum_{i=1}^{2n_c} \ln(s_i), \tag{B1}$$

subject to equality ($\boldsymbol{h}_{\mathrm{e}}$), inequality ($\boldsymbol{h}_{\mathrm{ie}}$) and decision variable bounds ($\boldsymbol{b}$) constraints, respectively:

$$\boldsymbol{h}_{\mathrm{e}}(\boldsymbol{D}, \boldsymbol{S}) = \boldsymbol{0}; \tag{B2}$$

$$\boldsymbol{h}_{\mathrm{ie}}(\boldsymbol{D}, \boldsymbol{S}) + \boldsymbol{s}_{\mathrm{ie}} = \boldsymbol{0}; \ \ \boldsymbol{s}_{\mathrm{ie}} \ge \boldsymbol{0}; \tag{B3}$$

$$\boldsymbol{b}(\boldsymbol{D}) + \boldsymbol{s}_{\mathrm{b}} = \boldsymbol{0}; \ \ \boldsymbol{s}_{\mathrm{b}} \ge \boldsymbol{0}. \tag{B4}$$

In (B2) are represented equality constraints, such as active and reactive power balance equations and fixed exchange equality constraints, defined by (A2a, b), (A3a, b) and (A6), respectively. In (B3) are represented inequality constraints, such as limits on active power branch flow and voltage magnitude, defined by (A4) and (A5), respectively. In (B4) are represented inequality (lower/upper bounds) constraints, such as limits on the amount of active and reactive power produced at generator buses and limits on the active power consumed at load buses, defined by (A7a-d).

The Lagrangian function of this particular OPF model from (B1)-(B4) is formulated as:

$$\Im = f(\boldsymbol{D}) - \mu \sum_{i=1}^{2n_c} \ln(s_i) - \boldsymbol{\lambda}^{\mathrm{T}}[-\boldsymbol{h}_{\mathrm{e}}(\boldsymbol{D}, \boldsymbol{S})] - \boldsymbol{\mu}^{\mathrm{T}}[-\boldsymbol{h}_{\mathrm{e}}(\boldsymbol{D}, \boldsymbol{S}) - \boldsymbol{s}_{\mathrm{ie}}]$$
$$- \boldsymbol{\pi}^{\mathrm{T}}[-\boldsymbol{b}(\boldsymbol{D}) - \boldsymbol{s}_{\mathrm{b}}]. \tag{B5}$$

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