A Novel Target Tracking Algorithm for Simultaneous Measurements of Radar and Infrared Sensors

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***Abstract*—In this paper, a game theory filtering technique is proposed to track a maneuvering target using radar/infrared (IR) sensors. It is shown that use of game theory technique can improve filter performance in presence of model uncertainties, measurement noise, and unknown steering command of the target. The tracking problem of maneuvering target is formulated as a zero-sum dynamic game and a utility function is developed to find equilibrium point of this game in a deterministic fashion to estimate target characteristics, including its position and velocity. To improve the filter performance, a proposed linear matrix inequality is implemented to obtain the introduced parameter in utility function. The robustness of the filter is guaranteed by minimizing the utility function for the worst case region of the measurement noise and steering command. Simulation results illustrate the improved performance of the proposed filter compared to extended Kalman and cubature Kalman filters.**

***Index Terms*—infrared sensors, radar tracking, state estimation, filtering algorithms, minimax techniques.**

I. INTRODUCTION

Target tracking is the most fundamental task associated with radar, infrared, and optical target tracking systems. The problem of target tracking is a Bayesian filtering problem to estimate characteristics of the target such as position, velocity, and acceleration in 3D Cartesian coordinate. Nowadays, radar/IR tracking system is interesting to researchers due to its flexibility, redundancy, simplicity, and reliability [1-5]. The radar is an active sensor that uses radio waves to determine range and azimuth angles of the target. However, it is easy to be interfered by electromagnetic signals. These unwanted signals may originate from internal and external sources, both passive and active. The IR sensor is a passive sensor, which has no effect on electromagnetic interference and is sensitive to atmospheric conditions [6-7]. Therefore, the target tracking task could considerably improve tracking precision by using radar and IR sensors. However, the measurements of radar and IR sensor are nonlinear. Therefore, the filter for maneuvering target tracking should be studied for radar/IR target tracking systems.

Model uncertainties, measurement noise, and unknown steering command of the target are some of the challenging problems in target tracking systems [8-12]. In [13], an ant colony estimator is proposed to track manevarable target. Evolutionary algorithms [14-17] are rarely utilized in target tracking systems. Minimax algorithms are utilized by some researchers to track manevarable target [18-19]. In [18-19], the minimax algorithm is used to minimize estimation error by assuming worst case conditions for system and measurement noise sequences. In [20], the target tracking problem is formulated as zero-sum game and a minimax algorithm is developed to estimate target position in sensor networks. In [21], an optimal minimax filter is proposed for tracking a target based on the relationship between the measurements and states. In [22], the problem of H_∞ optimal state estimation filter of linear continuous-time system is evaluated when target tracking problem is formulated as a stochastic game. A reduced minimax state estimation filter is proposed in [23] and a version of minimax state estimation filter is developed for singular linear stationary continues-time dynamic systems in [24]. A discreet-time version of the state estimation filter used in [23] has been developed in [25]. In [23],[25], no constrains are assumed for the state variables. The constraints on the state variables are commonly neglected in state estimation filter design procedure. To extend minimax filter for constrained state variables of linear discrete-time dynamic systems, a constrained state estimation filter is developed in [26]. All proposed algorithms assume that the measurement matrix is constant and consequently they cannot be utilized in radar/IR target tracking systems. To resolve this limitation, a discrete-time game theory technique is used in this paper.

With these descriptions, the contributions of this paper are twofold. First, the target tracking problem is formulated as a discrete-time zero-sum game. Then, similar to [20], a state estimation algorithm is developed based on game to be utilized in radar/IR target tracking systems with a simultaneous update algorithm. The solution of the game is developed by algebraic methods and an iterative algorithm is developed to accurately estimate of the position and velocity of maneuverable target in 3D Cartesian coordinate. Clearly, the proposed algorithm is a robust minimax iterative filter because it is obtained by minimizing estimation error under worst possible steering command.

This paper organized as follows. In section II, after describing and formulating the problem, a discrete-time zero-sum game is introduced for the target tracking problem. In the following, the problem is solved and its optimality is shown. Also, a Linear Matrix Inequality (LMI) relation is suggested to compute a parameter of filter and improve the filter performance. In section III, to evaluate performance of the proposed filter, different scenarios are simulated and the results are compared to Extended Kalman Filter (EKF) and

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Cubature Kalman Filter (CKF). Finally, the paper is summarized in the last section.

II. Problem Formulation

Most tracking algorithms are model based since knowledge of target motions is available in many applications. Clearly, a good model selection facilitates target tracking task. The most common model for the maneuverable target that is used in literature is white-noise acceleration model and it assumes that the target acceleration vector is zero-mean white noise process. This is in fact the model that is used in this paper in order to be able to compare the results of the proposed filter with the existing simulations. The discrete-time white-noise acceleration dynamic model that is used here is of the following form:

$$\begin{cases} \bar{x}_{k+1} = A\bar{x}_k + B\nu_k + \widetilde{B}\mu_k \\ y_k = f(\bar{x}_k) + \omega_k \end{cases} \tag{1}$$

where

$$\bar{x}_k = \begin{bmatrix} x_k & \dot{x}_k & \ddot{x}_k & y_k & \dot{y}_k & \ddot{y}_k & z_k & \dot{z}_k & \ddot{z}_k \end{bmatrix}^T \tag{2}$$

$$A = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & A_1 \end{bmatrix},\ A_1 = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$B = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_1 & 0 \\ 0 & 0 & B_1 \end{bmatrix},\ B_1 = \begin{bmatrix} T^2/2 \\ T \\ 1 \end{bmatrix} \tag{4}$$

In (1), k is the scan time index, T is the sample interval, $\bar{x}_k \in R^n$ is the system state vector at the k-th step consisting of position, velocity, and acceleration vectors in 3D Cartesian coordinate given by (2), $y_k \in R^r$ is the measurement vector, $\nu_k \in R^m$ is process noise vector, $\omega_k \in R^r$ is measurement noise vector, μ_k is the adversary steering command vector that is of course unknown to us. A and B are matrices given by (3) and (4) respectively, and $\widetilde{B}$ is a matrix with appropriate dimension relating state of the system to steering command of the maneuvering target. $f(\bullet)$ is measurement model that is a nonlinear function and is defined later. It is assumed that ν_k and ω_k are white noise processes that are mutually uncorrelated and satisfy (5) and (6) for each i and j respectively. In (5) and (6), $E(\bullet)$ is the expected value and δ_{ij} is Kronecker delta function. In general, μ_k is selected such that the worst case scenario for state estimation is obtained. According to [27], this worst case is achieved when μ_k is given by (7).

$$E(\nu_i) = 0, E(\nu_i \nu_j^T) = R\delta_{ij} \tag{5}$$

$$E(\omega_i) = 0, E(\omega_i \omega_j^T) = M\delta_{ij} \tag{6}$$

$$\mu_k = \widetilde{K}_a\left(G_k\left(\bar{x}_k - \hat{\bar{x}}_k\right) + n_k\right) \tag{7}$$

In (7), $\widetilde{K}_a$ is adversary gain matrix that will be calculated at each step, $\hat{\bar{x}}_k$ is an estimate of $\bar{x}_k$ as (8), G_k is a given matrix, and n_k is white noise process that satisfies (9). It is assumed that ν_k, ω_k, n_k, and initial condition $\bar{x}_0$ are mutually uncorrelated.

$$\hat{\bar{x}}_k = \begin{bmatrix} \hat{x}_k & \hat{\dot{x}}_k & \hat{\ddot{x}}_k & \hat{y}_k & \hat{\dot{y}}_k & \hat{\ddot{y}}_k & \hat{z}_k & \hat{\dot{z}}_k & \hat{\ddot{z}}_k \end{bmatrix}^T \tag{8}$$

$$E(n_i) = 0, E(n_i n_j^T) = N\delta_{ij} \tag{9}$$

Clearly, $f(\bullet)$ in (1) depends on the type of sensors. As stated earlier, in this work combination of radar and IR sensors has been considered for tracking a maneuvering target. It is assumed that radar and IR sensors lie in the same platform. Therefore it can be assumed that the two sensors locate in the same position. By considering a maneuverable target in 3D Cartesian coordinate, the measuring geometry relationship between target and sensors platform is shown in Fig. 1.

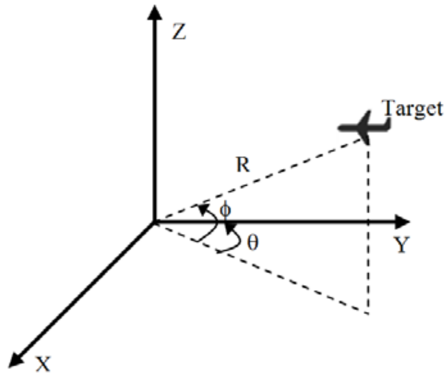

Figure 1. Measuring geometry relationship between target and sensors platform

The measurement model for the 2D radar sensor is as (10) where R_k^R is the range and θ_k^R is the azimuth angle given by (11) and (12) respectively, and ω_k^{Radar} is the measurement noise vector for the radar sensor.

$$z_k^R = h_{Radar}(\bar{x}_k) = \begin{bmatrix} R_k^R \\ \theta_k^R \end{bmatrix} + \omega_k^{Radar} \tag{10}$$

$$R_k^R = \sqrt{x_k^2 + y_k^2 + z_k^2} \tag{11}$$

$$\theta_k^R = \tan^{-1}\left(y_k / x_k \right) \tag{12}$$

The measurement model for the IR sensor is as (13) where θ_k^I is the azimuth angle and ϕ_k^I is the elevation angle given by (14) and (15) respectively, and ω_k^{IR} is measurement noise vector for the IR sensor.

$$z_k^{IR} = h_{IR}(\bar{x}_k) = \begin{bmatrix} \theta_k^I \\ \phi_k^I \end{bmatrix} + \omega_k^{IR} \tag{13}$$

$$\theta_k^I = \tan^{-1}\left(y_k / x_k \right) \tag{14}$$

$$\phi_k^I = \tan^{-1}\left(z_k \Big/ \sqrt{{x_k}^2 + {y_k}^2} \right) \tag{15}$$

Considering all the above, $f(\bar{x}_k)$ can be written as:

$$f(\bar{x}_k) = \begin{bmatrix} z_k^R \\ z_k^{IR} \end{bmatrix} \tag{16}$$

Since in target tracking systems unbiased state estimation filters are more suitable compared to biased state estimation filters, the filter structure is assumed as (17) with zero mean initial condition where K_f is filter gain matrix and $\hat{\hat{y}}_k$ is given by (18). In addition to mathematical tractability of an unbiased filter, the mean value of an unbiased filter equals the true value of the quantity that must be estimated.

$$\hat{\hat{x}}_{k+1} = A\hat{\hat{x}}_k + K_f\left(y_k - \hat{\hat{y}}_k\right) \tag{17}$$

$$\hat{\hat{y}}_k = f\left(\hat{\hat{x}}_k\right) \tag{18}$$

Let $K_a = \widetilde{B}\widetilde{K}_a$; $G_k = C_k$; $F_k = A - K_f C_k + K_a C_k$; and having defined the above, the estimation error can be defined as:

$$e_{k+1} = F_k e_k + B\nu_k + K_a n_k - K_f \omega_k \tag{19}$$

where C_k is the Jacobian of $f\left(\hat{\hat{x}}_k\right)$. In order to define a zero-sum game, the estimation error given by (19) is decomposed as follows:

$$e_k = e_k^f + e_k^a \tag{20}$$

Furthermore, e_k^f and e_k^a update equations can be written as:

$$e_{k+1}^f = F_k e_k^f + B\nu_k - K_f \omega_k \tag{21}$$

$$e_{k+1}^a = F_k e_k^a + K_e n_k \tag{22}$$

A. Discrete-Time Zero-Sum Dynamic Game

In order to determine K_f and K_a matrices, a discrete-time dynamic game must be defined. The filter algorithm as pursuer looks to obtain a K_f sequence that minimizes estimation error and in opposition the target as adversary looks to find a K_a sequence that maximizes estimation error. Consequently, the proposed utility function can be written as:

$$J_N\left(K_f, K_a\right) = trace\left\{\sum_{k=0}^{N-1} W_k E\left\{\left\|e_k^f\right\|^2 - \gamma_k{}^2\left\|e_k^a\right\|^2\right\}\right\} \tag{23}$$

In (23), the scalar $\gamma_k > 0$ is an adjustable parameter that is necessary when $f\left(\hat{\hat{x}}_k\right)$ has high nonlinearity, N is the time horizon and W_k is any positive definite weighting matrix. It can be shown that when γ_k is set to zero in the above equation; the solution will converge towards that of Extended Kalman Filter (EKF). Note that when K_f^* and K_a^* are the solutions of the zero-sum game, then the following inequality is satisfied.

$$J_N\left(K_f^*, K_a\right) \le J_N\left(K_f^*, K_a^*\right) \le J_N\left(K_f, K_a^*\right) \tag{24}$$

B. The Game Solution and the State Estimation Filter

The solution of the zero-sum game is now discussed.

Lemma 1: the cost function $J_N\left(K_f, K_a\right)$ can be written as:

$$J_N\left(K_f, K_a\right) = trace\left\{\sum_{k=0}^{N-1} W_k Q_k\right\} \tag{25}$$

where $Q_k \in R^{n\times n}$ is symmetric and satisfies (26). In (26), R, M and N matrices are defined in (5), (6) and (9) respectively.

$$Q_{k+1} = F_k Q_k F_k^T + BRB^T + K_f M K_f^T - \gamma_k^2 K_a N K_a^T \tag{26}$$

Proof: denoting $Q_k^f = E\left(e_k^f {e_k^f}^T\right)$ and $Q_k^a = E\left(e_k^a {e_k^a}^T\right)$ and using assumption that that v_k, ω_k, n_k and initial condition x_0 are mutually uncorrelated, one can write

$$Q_{k+1}^f = F_k Q_k^f F_k^T + BRB^T + K_f M K_f^T \tag{27}$$

$$Q_{k+1}^a = F_k Q_k^a F_k^T + \gamma_k^2 K_a N K_a^T \tag{28}$$

Defining $Q_k = Q_k^f - Q_k^a$ establishes the proof. Note that Q_k is a symmetric matrix. ■

Theorem 1: Assuming $Q_k \ge 0$, $C_k Q_k C_k{}^T + M \ge 0$, and $\gamma^2 N - C_k Q_k C_k{}^T \ge 0$ for $k = 1,2,....,N-1$, the optimal solutions of K_f and K_a are given by:

$$K_f^* = AP_k C_k{}^T M^{-1} \tag{29}$$

$$K_a^* = \gamma_k^{-2} AP_k C_k{}^T N^{-1} \tag{30}$$

$$P_k^{-1} = Q_k^{-1} + C_k{}^T\left(M^{-1} - \gamma_k^{-2} N^{-1}\right)C_k \tag{31}$$

Proof: from the result of Lemma 1, one can see that the first derivative of $trace\left(Q_{k+1}\right)$ is zero at K_f^* and K_a^*. Therefore, the following relations must be satisfied:

$$\left.\frac{\partial trace\left(Q_{k+1}\right)}{\partial K_f}\right|_{K_f = K_f^*} = 0 \tag{32}$$

$$\left.\frac{\partial trace\left(Q_{k+1}\right)}{\partial K_a}\right|_{K_a = K_a^*} = 0 \tag{33}$$

Using (32) and (33) one can write

$$\begin{cases} K_f^*\left(C_k Q_k C_k{}^T - M\right) - K_a^*\left(C_k Q_k C_k{}^T\right) = AQ_k C_k{}^T \\ K_f^*\left(C_k Q_k C_k{}^T\right) - K_a^*\left(C_k Q_k C_k{}^T - \gamma_k^2 N\right) = AQ_k C_k{}^T \end{cases} \tag{34}$$

By defining $\mathrm{K} = \left[K_f^* \quad -K_a^*\right]$, $\psi = \begin{bmatrix} C_k \\ C_k \end{bmatrix}$, and $\Lambda = \begin{bmatrix} R & 0 \\ 0 & -\gamma_k{}^2 N \end{bmatrix}$, the above equation can be written in the following matrix form:

$$\mathrm{K}\Psi Q_k \Psi^T + \mathrm{K}\Lambda = AQ_k \psi^T \tag{35}$$

From the above equation, matrix K is obtained as:

$$\mathrm{K} = AQ_k \Psi'\left(\Psi Q_k \Psi' + \Lambda\right)^{-1} \tag{36}$$

Using matrix inversion lemma as [28], the above can be rewritten as:

$$\mathrm{K} = A\left(Q_k^{-1} + \Psi\Lambda^{-1}\Psi^T\right)^{-1}\Psi^T\Lambda^{-1} \tag{37}$$

Note that (37) is nothing but (29), (30) and (31).

To ensure that K_f and K_a gain sequences minimize and maximize estimation error respectively, the $trace\left(Q_{k+1}\right)$ must be quadratic and convex with respect to K_f and K_a. In other words, the second derivatives of $trace\left(Q_{k+1}\right)$ with respect to K_f and K_a must satisfy the following relations.

$$\frac{\partial^2 trace\left(Q_{k+1}\right)}{\partial^2 K_f} \ge 0; \frac{\partial^2 trace\left(Q_{k+1}\right)}{\partial^2 K_a} \le 0 \tag{38}$$

But notice that

$$\frac{\delta^2\, trace(Q_{k+1})}{\delta^2 K_p} \geq 0 \Rightarrow C_k Q_k C_k{}^T + M \geq 0 \ \ (a);$$
$$\frac{\delta^2 trace(Q_{k+1})}{\delta^2 K_e} \leq 0 \Rightarrow \gamma_k^2 N - C_k Q_k C_k{}^T \geq 0 \ \ (b) \qquad (39)$$

The parameter γ_k can be chosen such that (39)-b is satisfied. ■

Using the results of the above theorem and the fact that the covariance matrix is given by

$$Q_{k+1} = F_k Q_k F_k^T + BRB^T + K_f M K_f^T - \gamma_k^2 K_a N K_a^T \qquad (40)$$

one can use the following relations recursively in order to obtain state estimates assuming that matrix C_k which is the Jacobian of $f\left(\hat{\bar{x}}_k\right)$ is given.

$$\hat{\bar{x}}_{k+1} = A\hat{\bar{x}}_k + K_f\left(y_k - \hat{\bar{y}}_k\right)$$
$$P_k^{-1} = Q_k^{-1} + C_k^T\left(M^{-1} - \gamma_k^{-2} N^{-1}\right)C_k$$
$$Q_{k+1} = F_k Q_k F_k^T + BRB^T + K_f M K_f^T - \gamma_k^2 K_a N K_a^T \qquad (41)$$
$$K_f = A P_k C_k^T M^{-1}$$
$$K_a = \gamma_k^{-2} A P_k C_k^T N^{-1}$$

Remark: our main contribution is development a filtering technique to minimize the cost function, (25), under worst possible steering command. The optimal solution of the filter gain can be obtained by (29). It can be seen that the filter gain is obtained by minimizing the cost function.

To obtain C_k, Jacobians of (10) and (13) are used to obtain the corresponding measurement matrices as given by (42) and (43). In (42) and (43), $\hat{R}_k$, $\hat{\theta}_k$ and $\hat{\phi}_k$ are given by (44), (45) and (46) respectively.

$$C_k^{Radar} = \frac{\partial z_k^R}{\partial \bar{x}_k}\bigg|_{\bar{x}_k = \hat{\bar{x}}_k} \qquad (42)$$

$$C_k^{IR} = \frac{\partial z_k^{IR}}{\partial \bar{x}_k}\bigg|_{\bar{x}_k = \hat{\bar{x}}_k} \qquad (43)$$

$$\hat{R}_k = \sqrt{\hat{x}_k^2 + \hat{y}_k^2 + \hat{z}_k^2} \qquad (44)$$

$$\hat{\theta}_k = \tan^{-1}\left(\hat{y}_k / \hat{x}_k\right) \qquad (45)$$

$$\hat{\phi}_k = \tan^{-1}\left(\hat{z}_k \Big/ \sqrt{\hat{x}_k{}^2 + \hat{y}_k{}^2}\right) \qquad (46)$$

Combing the above two equations, matrix C_k is obtained as follows:

$$C_k = J_f = \begin{bmatrix} C_k^{Radar} \\ C_k^{IR} \end{bmatrix} \qquad (47)$$

Clearly, $C_k = J_f$ is a time varying matrix. The measurement noise used in (1), ω_k, is defined as:

$$\omega_k = \begin{bmatrix} \omega_k^{Radar} \\ \omega_k^{IR} \end{bmatrix} \qquad (48)$$

where ω_k^{Radar} and ω_k^{IR} are as (10) and (13), respectively. It only remain to show how to obtain γ_k. The proposed approach to compute near optimal γ_k is based on LMI. This parameter is chosen such that the trace of the covariance matrix, $trace(Q_{k+1})$, is as small as possible at every step. This problem is considered as a generalized eigenvalue problem where the objective is to limit the eigenvalues of a positive definite matrix to be as small as possible subject to an LMI constraint. The proposed LMI relation is as follows:

minimize λ

$$\textbf{\textit{subject to}} \begin{bmatrix} \lambda I - Q_k & 0 \\ 0 & \gamma_k^2 N - C_k Q_k C_k^T \end{bmatrix} > 0 \qquad (49)$$

III. Simulation Results

To illustrate the performance of the proposed filter, different scenarios are considered. We consider four tracking scenarios with varying measurement noise and evaluate the performance of the Game Theory Based Filter (GTBF) for these scenarios. Simulations are performed using Matlab software. Simulation setup and simulation results are discussed next.

A. Simulation Setup

It is assumed that the radar and IR sensors are located at the origin of the 3D Cartesian coordinate. Also, for each simulation, the number of sensors is two, one for each type of sensor. The sampling rate is $T=0.5$ second. The target dynamics can be approximated by discrete-time white-noise acceleration dynamic model as (1). Each measurement, $y_k = \left[y_k^1, y_k^2\right]$ is a 4-dimentional vector given by:

$$y_k^1 = z_k^R = \begin{bmatrix} R_k^R \\ \theta_k^R \end{bmatrix} + \omega_k^{Radar} \qquad (50)$$

$$y_k^2 = z_k^{IR} = \begin{bmatrix} \theta_k^I \\ \phi_k^I \end{bmatrix} + \omega_k^{IR} \qquad (51)$$

The normal measurement noise vector for the radar sensor is a zero mean Gaussian sequence with standard deviation of 20 m for range and 10 milliradian (mrad) for azimuth angle. The normal measurement noise vector for IR sensor is a zero mean Gaussian sequence with standard deviation of 7 mrad for both azimuth and elevation angles. Therefore, the measurement noise covariance matrix is a diagonal matrix as (52). In the simulation, the covariance matrices corresponding to initial state, process noise, and n_k are given by (53), (54), and (55) respectively.

$$M = diag\left(\left[20^2 \ \ 1\times10^{-4} \ \ 4\times10^{-4} \ \ 4\times10^{-4}\right]\right) \qquad (52)$$
$$Q_0 = diag\left(\left[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\right]\right) \qquad (53)$$
$$R = diag\left(\left[2^2 \ \ 2^2 \ \ 1.5^2\right]\right) \qquad (54)$$
$$N = diag\left(\left[30^2 \ \ 5\times10^{-3} \ \ 3.5\times10^{-3} \ \ 3.5\times10^{-3}\right]\right) \qquad (55)$$

Root Mean Square Error (RMSE) and Cumulative Root Mean Square Error (CRMSE) frequently used as performance indexes in target tracking systems. RMSE represents the standard deviation of the differences between estimated and actual values. The RMSE of position and velocity estimates as (56) and (57) are used in this paper to illustrate the performance of the proposed filter. Also, CRMSE of position and velocity estimates are as (58) and (59) respectively. In each scenario, the averaged results based on 50 Monte-Carlo iterations are illustrated.

$$RMSE_P(k)=\sqrt{\frac{1}{3}\left((x_k-\hat{x}_k)^2+(y_k-\hat{y}_k)^2+(z_k-\hat{z}_k)^2\right)} \tag{56}$$

$$RMSE_V(k)=\sqrt{\frac{1}{3}\left(\left(\dot{x}_k-\hat{\dot{x}}_k\right)^2+\left(\dot{y}_k-\hat{\dot{y}}_k\right)^2+\left(\dot{z}_k-\hat{\dot{z}}_k\right)^2\right)} \tag{57}$$

$$CRMSE_P(K)=\frac{1}{KT}\sum_{k=1}^{K}\sqrt{\frac{k}{3K}\left((x_k-\hat{x}_k)^2+(y_k-\hat{y}_k)^2+(z_k-\hat{z}_k)^2\right)} \tag{58}$$

$$CRMSE_V(K)=\frac{1}{KT}\sum_{k=1}^{K}\sqrt{\frac{k}{3K}\left(\left(\dot{x}_k-\hat{\dot{x}}_k\right)^2+\left(\dot{y}_k-\hat{\dot{y}}_k\right)^2+\left(\dot{z}_k-\hat{\dot{z}}_k\right)^2\right)} \tag{59}$$

B. First Scenario

In first scenario, simulations are performed with nominal noise statistics of radar/IR sensors. The target starts at [10500 8300 1150] in Cartesian coordinate in meters. Also, the initial velocity is [-50 -200 -100] m/s. The target stays at constant velocity between 0 to 10 seconds, a constant acceleration of [-2 1.5 0] m/s^2 between 10 to 25 seconds, a constant acceleration of [2 -2 0] m/s^2 between 25 to 32 seconds, a constant acceleration of [3 -2 0] m/s^2 between 32 to 50 seconds, and a constant velocity between 50 to 60 seconds.

The initial state of the tracking algorithms $\hat{x}_0$ can be calculated by using two-step extrapolation algorithm and coordinate transform of the measurements. The actual and the estimated trajectory using the proposed filter are shown in Fig. 2. It can be seen that the estimated trajectory has suitable performance in this scenario. Fig. 3 and Fig. 4 display RMSE of position and velocity for GTBF, EKF, and CKF [29] respectively. Although the GTBF results in more accurate estimates, results show that all filters can estimate the position and velocity of maneuverable target in 3D Cartesian coordinate with reasonable accuracy. The GTBF results in position errors that average around 27.77 m, while the EKF and CKF give position errors that average about 30.95 m and 29.54 m respectively. The GTBF velocity error averages around 8.85 m/s, while the EKF and CKF give velocity errors that average about 10.9 m/s and 10.95 m/s respectively. CRMSE and operation time test for three filters are shown in Table I. It can be seen that The $CRMSE_p$ of GTBF is reduced by 6% for EKF and 3% for CKF. Also, the $CRMSE_v$ of GTBF is also reduced by 11% for both EKF and CKF. The operation time of the CKF is longer than GTBF and EKF.

TABLE I. CRMSE AND RUNTIME COMPARISON OF THREE DIFFERENT FILTERS FOR FIRST SCENARIO

Filter	Position (*m*)	Velocity (*m/s*)	Time (*s*)
GTBF	3.8703	2.2807	1.427
EKF	4.1141	2.5351	0.86868
CKF	3.9887	2.5412	10.2603

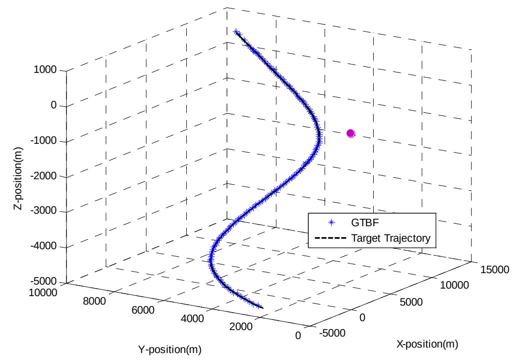

Figure 2. Position estimation in first scenario

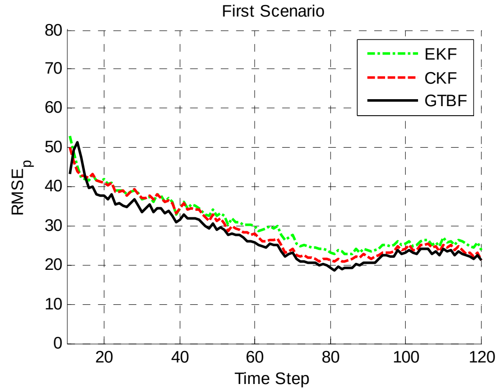

Figure 3. RMSE of position in first scenario

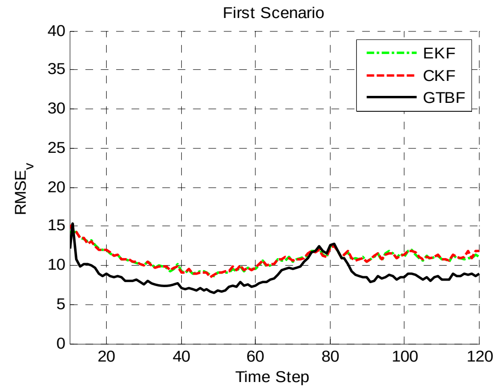

Figure 4. RMSE of velocity in first scenario

C. Second Scenario

In second scenario, simulations are performed with off-nominal noise statistics. The target trajectory is the same as first scenario. Also, all parameters of the filters are the same as first scenario. Simulation results are shown in Fig. 5 and Fig. 6 when measurement noise vector for the radar sensor is a zero mean Gaussian sequence with standard deviation of 40 m for range and 20 mrad for azimuth angle, and measurement noise vector for IR sensor is a zero mean Gaussian sequence with standard deviation of 14 mrad for both azimuth and elevation angles. Fig. 5 and Fig. 6 display RMSE of position and velocity for GTBF, EKF, and CKF respectively. Simulation results show that GTBF outperforms both EKF and CKF. The GTBF results in position errors that average around 55.39 m, while the EKF and CKF give position errors that average about 66.81 m and 67.14 m respectively. The GTBF velocity error averages around 16.38 m/s, while the EKF and CKF give velocity errors that average about 31.38 m/s and 31.37 m/s respectively. Also, *CRMSE* of state estimation and operation time test for three filters are shown in Table II. It can be seen that the $CRMSE_p$ of GTBF is reduced by 10% for both EKF and CKF. The $CRMSE_v$ of GTBF is also reduced by 38% for both EKF and CKF.

TABLE II. CRMSE AND RUNTIME COMPARISON OF THREE DIFFERENT FILTERS FOR SECOND SCENARIO

Filter	Position (*m*)	Velocity (*m/s*)	Time (*s*)
GTBF	5.4807	3.1239	1.4999
EKF	6.0368	4.3229	0.86413
CKF	6.0346	4.3278	9.9894

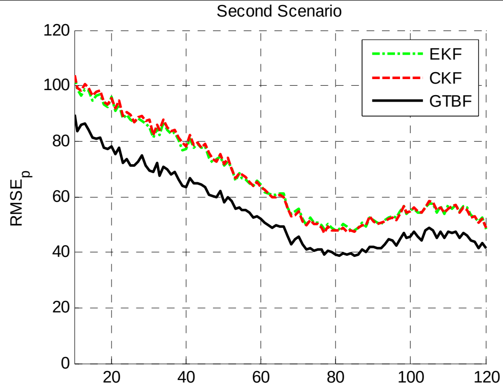

Figure 5. RMSE of position in second scenario

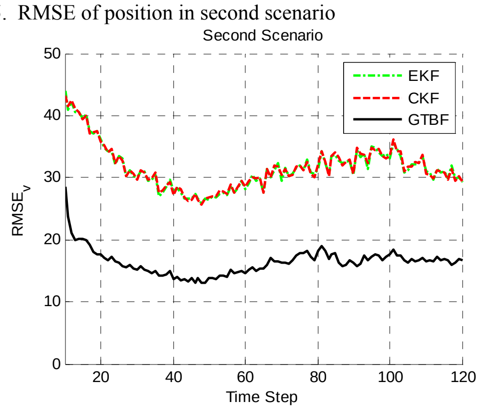

Figure 6. RMSE of velocity in second scenario

From the discussion above, one can infer that GTBF, EKF and CKF can estimate the position and velocity of maneuverable target in 3D Cartesian coordinate with reasonable accuracy when the noise statistics are nominal. However, if the noise statistics are not known, then the GTBF will perform better than the EKF and CKF.

D. *Third Scenario*

In the third scenario, simulations are performed with nominal noise statistics of radar/IR sensors. In this scenario, the target starts at [-1000 500 8000] in Cartesian coordinate in meters, and the target has an elliptic trajectory with constant altitude above the sensors. The model of the trajectory is given by:

$$\begin{cases} x_{k+1} = x_k + \left(500\cos(0.15kT)\right)T \\ y_{k+1} = y_k + \left(500\sin(0.15kT)\right)T \end{cases} \tag{60}$$

The actual and the estimated trajectory using the proposed filter are shown in Fig. 7. It can be seen that the proposed filter has an appropriate efficiency in this scenario. Fig. 8 and Fig. 9 display RMSE of position and velocity for GTBF, EKF, and CKF respectively. Clearly, the EKF and CKF have considerable estimation errors in comparison with GTBF. The GTBF results in position errors that average around 18.15 m, while the EKF and CKF give position errors that average about 36.97 m and 36.03 m respectively (almost twice as much or more). The GTBF velocity error averages around 12.86 m/s, while the EKF and CKF give velocity errors that average about 34.82 m/s and 34.01 m/s respectively. CRMSE and operation time test for three filters are shown in Table III. It can be seen that $CRMSE_p$ of GTBF (given by (58)) is reduced by 30% for EKF and 28% for CKF. As mentioned before, the operation time of the CKF is longer than GTBF and EKF.

TABLE III. CRMSE AND RUNTIME COMPARISON OF THREE DIFFERENT FILTERS FOR THIRD SCENARIO

Filter	Position (*m*)	Velocity (*m/s*)	Time (*s*)
GTBF	3.1251	2.2732	0.5241
EKF	4.1452	4.2893	0.4952
CKF	4.1265	4.0956	4.2512

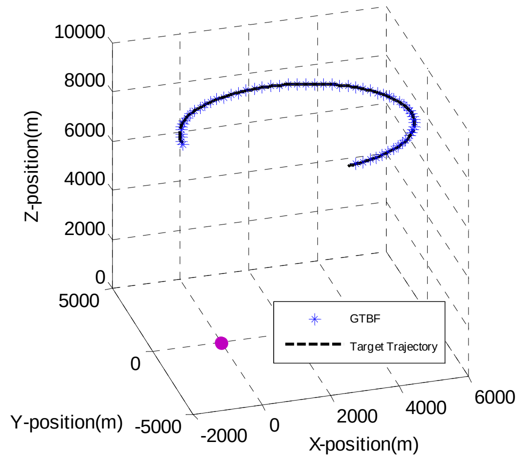

Figure 7. Position estimation in third scenario

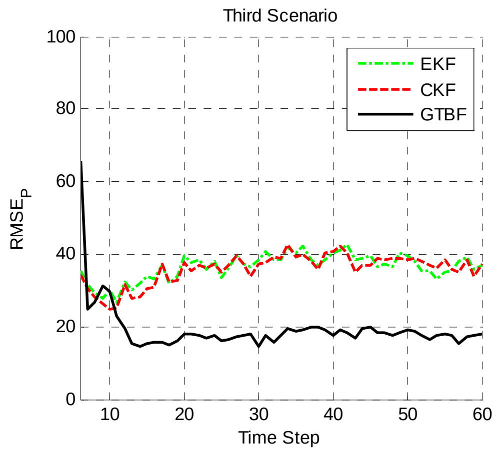

Figure 8. RMSE of position in third scenario

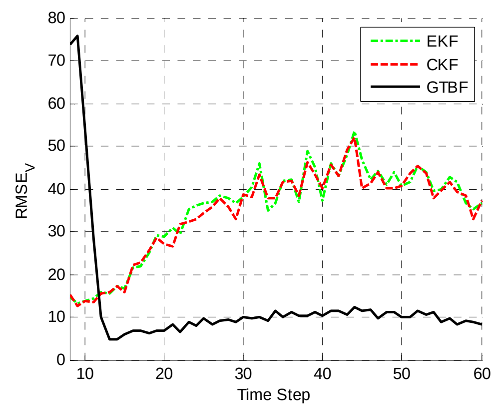

Figure 9. RMSE of velocity in third scenario

E. *Fourth Scenario*

In the fourth scenario, the target trajectory is the same as third scenario. The simulations are performed with off-nominal noise statistics. Simulation results are shown in Fig. 10 and Fig. 11 when measurement noise vector for the radar sensor is a zero mean Gaussian sequence with standard deviation of 60 m for range and 30 mrad for azimuth angle, and measurement noise vector for IR sensor is a zero mean Gaussian sequence with standard deviation of 25 mrad for both azimuth and elevation angles. Fig. 10 and Fig. 11 display RMSE of position and velocity for GTBF, EKF, and CKF respectively. The GTBF results in position errors that average around 77.64 m, while the EKF and CKF give position errors that average about 166.32 m and 177.48 m respectively. The GTBF velocity error averages around 45.84 m/s, while the EKF and CKF give velocity errors that average about 170.93 m/s and 166.74 m/s respectively. Also, CRMSE of state estimation and operation time test for three filters are shown in Table IV. It can be seen that the $CRMSE_p$ of GTBF is reduced by 44% for both EKF and CKF. The $CRMSE_v$ of GTBF is also reduced by 52% for both EKF and CKF.

TABLE IV. CRMSE AND RUNTIME COMPARISON OF THREE DIFFERENT FILTERS FOR FOURTH SCENARIO

Filter	Position (*m*)	Velocity (*m/s*)	Time (*s*)
GTBF	9.1324	5.2651	0.5831
EKF	13.8333	11.2314	0.5178
CKF	13.9214	10.9125	4.3142

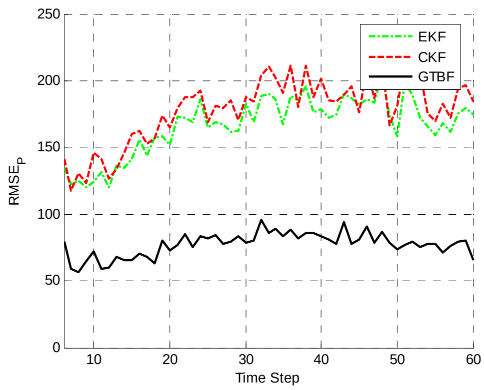

Figure 10. RMSE of position in fourth scenario

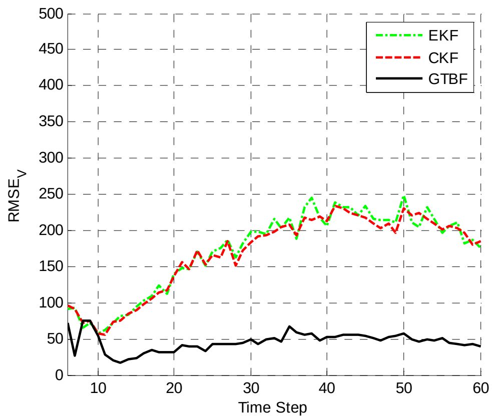

Figure 11. RMSE of velocity in fourth scenario

IV. CONCLUSION

A game theory based algorithm was developed for tracking a maneuvering target using radar/infrared (IR) sensors. The tracking problem in target tracking systems was formulated as a zero-sum game and equilibrium point of the game was derived to obtain the best estimate of position and velocity of target in 3D Cartesian coordinate. In order to prove the stability of the proposed filter, a parameter was introduced and LMI approach was used to calculate this parameter. One of the advantages of the proposed filter is that the filter gain is calculated by assuming worst possible unknown steering command of the target, and this makes the performance of the proposed filter less sensitive to steering command of the target. Simulations were performed for different scenarios and the results were compared to EKF and CKF. Simulation results illustrate significant improvement of the proposed filter compared to both EKF and CKF.

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