

A New Method for Maintaining Constant Dither Amplitude in Low Frequency PWM

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Abstract—Various controls for fluid flow and pressure are now required in related industries, and the pulse width modulation (PWM) and dithering techniques have become essential for the proportional control of solenoids. However, there is a fatal drawback when the dither current signals are generated as a by-product of low frequency PWM. That is, the average current and the dither amplitude in low frequency PWM cannot be controlled independently. Therefore, a new method for maintaining constant dither amplitudes is proposed in this paper. Throughout the mathematical analysis, the effect of PWM frequency and duty cycle on the average current and dither amplitude was investigated, and the analysis result was validated by electrical experiments. Based on the mathematical analysis, a new method that properly varies both the duty cycle and the PWM frequency to obtain the desired average current and constant dither amplitude was established and verified. This method requires only the calculations for determining the proper PWM frequency and duty cycle, so it is possible to improve the performance of a proportional solenoid valve without additional devices or cost.

Index Terms—pulse width modulation, circuit simulation, driver circuits, solenoids, mathematical analysis.

I. INTRODUCTION

A solenoid valve is the simplest device to control hydraulic motors, cylinders or large industrial valves, and has been used for a long time in the pneumatics and hydraulics fields. Solenoid valves can control the flow rate or pressure of fluids proportionally by shifting the internal spool position or the attraction force, and can allow fluids to flow by opening and closing the flow path. The former, the so-called proportional control, varies the solenoid input current with pulse width modulation (PWM), because the spool position and the attraction force are proportional to its current. The latter, called the on-off control switches two states of the flow path (on and off) by electric power through the solenoid. Various controls for fluid flow and pressure are now required in related industries, and the PWM technique, which is an effective way to control the electric current without resistors, has become essential for the proportional control of solenoids. In addition, the dithering technique has been applied to the proportional control in order to reduce the effects of stiction and hysteresis [1-10].

Dither is a small ripple in the solenoid current that causes a rapid and small movement of the spool around the desired position [1],[7-11]. It is intended to maintain consistent

motion with small oscillations, because the friction of a moving object is less than that of a stationary object. Because the PWM and dithering techniques are effective methods to control proportional solenoid valves (PSVs), they have been widely used in automotive components such as braking systems, automatic transmissions and diesel engine fuel flow control systems [5],[9],[13-16]. Furthermore, they have been employed in other industrial fields such as pneumatics, hydraulics and fluid flow control systems [1-4],[6-8],[11-12],[17-22]. These studies have focused on enhancing the dynamic performance and alleviating the hysteresis effect of PSVs, and high frequency PWM has been commonly employed to achieve them. Although several studies [5-8],[11] have used the dithering technique in low frequency PWM, high frequency PWM is mainly used because of a drawback of dithering in low frequency PWM.

There is a drawback when the dither current signals are generated as a by-product of low frequency PWM. The average current and dither amplitude in low frequency PWM cannot be controlled independently. This indicates that the conventional low frequency PWM technique, which varies only the duty cycle to change the current under a fixed PWM frequency, cannot maintain constant dither amplitude. If the dither amplitude is not maintained at a constant level, undesirably large oscillations could occur in the solenoid spool. Moreover, the solenoid spool could not respond to very small dither amplitude because of inertia. Accordingly, maintaining constant dither amplitude is essential for stable performance of PSVs.

Therefore, a new method for maintaining constant dither amplitudes is proposed in this paper. It properly varies both the duty cycle and the PWM frequency to obtain the desired average current and constant dither amplitude. Throughout the mathematical analysis, the effect of the PWM frequency and duty cycle on the average current and dither amplitude was investigated, and the analysis result was validated by electrical experiments. Then, the new method, which is capable of controlling the average current and the dither current amplitude separately in a limited range, was established, and the details for implementing the new method are presented.

II. MATHEMATICAL ANALYSIS

A. Dithering in Low Frequency PWM

Dither signals are classified into two categories; the dither in low and high frequency PWM. The dither signal in low

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frequency PWM, typically less than 1000 Hz, is caused by coil inductance inside a solenoid. Because inductance has the characteristic of opposing changes in current flow in an electric circuit, a time lag is introduced into the current as shown in Fig. 1(a). That is, the current gradually rises and falls by the PWM on-off state, and therefore the inherent current ripple (the so-called dither or parasitic dither) appears. Because the dither amplitude depends on the duty cycle, it is maximized at a 50% duty cycle and decreases to zero at 0% and 100% duty cycles. Because the dither current signal in low frequency PWM is generated as a by-product, the dither frequency is naturally equal to the PWM frequency. In dither signals, the average current and the dither amplitude indicate the arithmetic mean of the minimum and maximum current and the difference between them, respectively, as represented in Fig. 1(a). On the other hand, in high frequency PWM, typically above 5 kHz, there is little inherent current ripple as shown in Fig. 1(b), because of its fast on-off switching time. Thus, a separate dither signal is commonly superimposed on top of the current. The advantage of using high frequency dither is that the dither amplitude can be controlled independently as well as the average current, but an additional dither generator or electric circuit is required, increasing the cost of setup and maintenance. Unless a PSV is utilized in industrial applications requiring precise control, a device for high frequency PWM will become redundant.

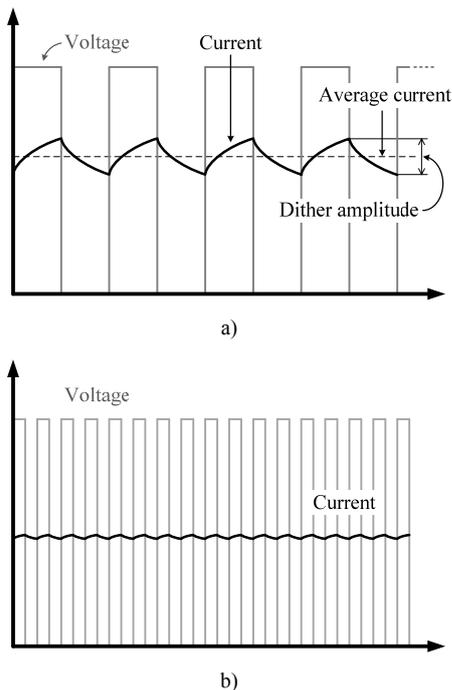


Figure 1. Comparison of the dither signals in a) low frequency PWM and b) high frequency PWM

B. Mathematical Model of PWM-Driven PSV

To investigate the effect of the PWM related variables, namely frequency (f_{PWM}) and duty cycle (D), on the average current (I_{avg}) and the dither amplitude (I_{amp}), a mathematical analysis was performed on a PWM-driven PSV. Because the electromagnetic coil inside a PSV can be considered equivalent to a resistor and an inductor connected in series, a PSV can be simplified as an RL circuit as depicted in Fig. 2 [7]. Herein, R and L denote coil resistance and coil inductance, individually. On the basis of Kirchoff's voltage

law, the current (i) can be obtained as

$$i(t) = \frac{v_s(t)}{R} \cdot (1 - e^{-\frac{t}{\tau}}) \quad (1)$$

Where v_s denotes the circuit voltage and the time constant (τ) is defined as L/R .

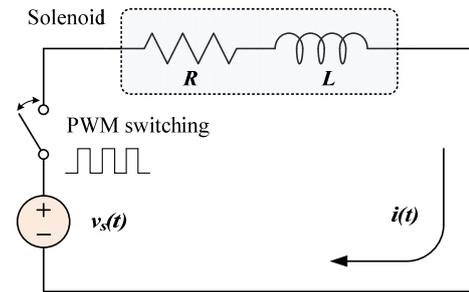


Figure 2. RL circuit equivalent to a PWM-driven PSV

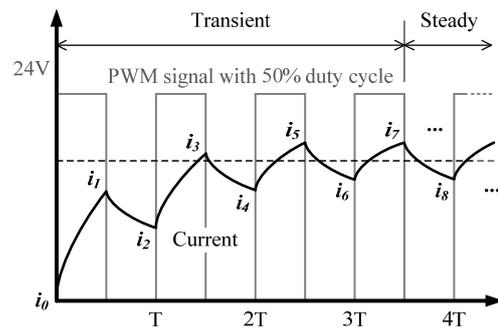


Figure 3. Current of a PSV by PWM signals

As shown in Fig. 3, the current level of a PSV gradually rises as soon as the electric power is supplied by PWM (transient state in terms of the average current), and the current oscillates periodically as the PWM is in progress (steady state). With regard to a PSV, the current charges during the PWM on-state and discharges during the off-state. When the initial current (i_0) is zero, the current at the charging ($i_m, m=1, 3, 5 \dots$) and discharging ($i_n, n=2, 4, 6 \dots$) states can be derived as

$$i_m = i_{m-1} \cdot e^{-\frac{DT}{\tau}} + \frac{v_s}{R} \cdot (1 - e^{-\frac{DT}{\tau}}) \quad (2)$$

$$i_n = i_{n-1} \cdot e^{-\frac{(1-D)T}{\tau}} \quad (3)$$

In these equations, the PWM period (T) is the reciprocal of the PWM frequency (f_{PWM}). By expanding the current equations in sequence and simplifying the mathematical expressions, the geometric progressions were discovered as seen in (4) and (5). Herein, the repetitive expressions can be replaced by α and β , and they are given by (6) and (7). The details of the expansion and simplification of the equations are well explained in [7].

$$i_m = \frac{v_s}{R} \cdot (1 - \alpha) \cdot \{(\alpha\beta)^p + (\alpha\beta)^{p-1} + \dots + (\alpha\beta) + 1\} \quad (4)$$

$$i_n = \frac{v_s}{R} \cdot (1 - \alpha) \cdot \beta \cdot \{(\alpha\beta)^q + (\alpha\beta)^{q-1} + \dots + (\alpha\beta) + 1\} \quad (5)$$

$$\alpha = e^{-\frac{DT}{\tau}} \quad (6)$$

$$\beta = e^{-\frac{(1-D)T}{\tau}} \quad (7)$$

By arranging the geometric series, the currents at charging (i_c) and discharging (i_d) in the steady state can be derived as

$$i_c = \frac{v_s}{R} \cdot \frac{(1-\alpha)}{(1-\alpha\beta)} \quad (8)$$

$$i_d = \frac{v_s}{R} \cdot \frac{(1-\alpha)}{(1-\alpha\beta)} \cdot \beta \quad (9)$$

Consequently, the average current (I_{avg}) and the dither amplitude (I_{amp}) in a PWM-driven PSV can be calculated as

$$I_{avg} = \frac{i_c + i_d}{2} = \frac{v_s}{R} \cdot \frac{(1-\alpha)}{(1-\alpha\beta)} \cdot \frac{(1+\beta)}{2} \quad (10)$$

$$I_{amp} = i_c - i_d = \frac{v_s}{R} \cdot \frac{(1-\alpha)}{(1-\alpha\beta)} \cdot (1-\beta) \quad (11)$$

From this mathematical model, it is found that the PWM-related variables (f_{PWM} , D) and the constants (R , L , v_s) have influence upon the average current and the dither amplitude simultaneously.

C. Effect of PWM Frequency and Duty Cycle on Average Current and Dither Amplitude

As shown in Fig. 4, the average current and dither amplitude according the PWM frequency and duty cycle were estimated from the mathematical model. The target PSV is the TS98-T34 model of HydraForce, whose coil resistance (R) and inductance (L) are 28.5 Ω and 330 mH, respectively, at 24 V (v_s) and 20 $^{\circ}\text{C}$. From the point of view of the PWM frequency, the average current (I_{avg}) is nearly steady regardless of the frequency, whereas the dither amplitude (I_{amp}) decreases negative exponentially as the PWM frequency increases. In terms of the duty cycle, meanwhile, the average current increases in proportion to the duty cycle, but the dither amplitude is maximized at 50% duty cycle and becomes zero at 0% and 100% duty cycles. That is, the dither amplitude is symmetric with respect to 50% duty cycle and there is no dither at 0% and 100% duty cycles.

This plot indicates that the average current and the dither amplitude are restricted in the conventional PWM method because it varies the duty cycle only at a fixed frequency. However, it also implies that they can be controlled independently within a limited range by adjusting both the PWM frequency and the duty cycle. To obtain the desired average current and the dither amplitude, therefore, the proper values of the PWM frequency and duty cycle must be chosen.

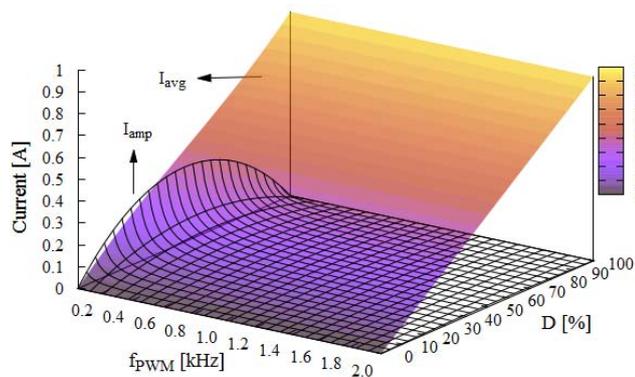


Figure 4. Average current and dither amplitude according to the PWM frequency and duty cycle

III. VALIDATION OF THE MATHEMATICAL ANALYSIS

A series of electrical experiments was carried out to validate the mathematical analysis. As described in Fig. 5(a),

the PSV (HydraForce; TS98-T34) and the peripheral electric system were prepared to measure the average current and dither amplitude. The N-channel MOSFET (Fairchild Semiconductor; IRF530A) switches a 24 V electric power supply along with the gate input of a 5 V PWM signal that was generated from the microprocessor (ATmega2560). The current sensor (Seri2B; SCV-D4-03H05-12D), located at the current input of the PSV, converts the current signal (0-3 A) into a voltage level (0-5 V). The oscilloscope (Agilent Technologies; DSO7054A) was used to display the current and the PWM waveform on the screen at the same time. Fig. 5(b) shows the PWM voltage and the current data acquired from the oscilloscope at a 200 Hz PWM frequency and 50% duty cycle.

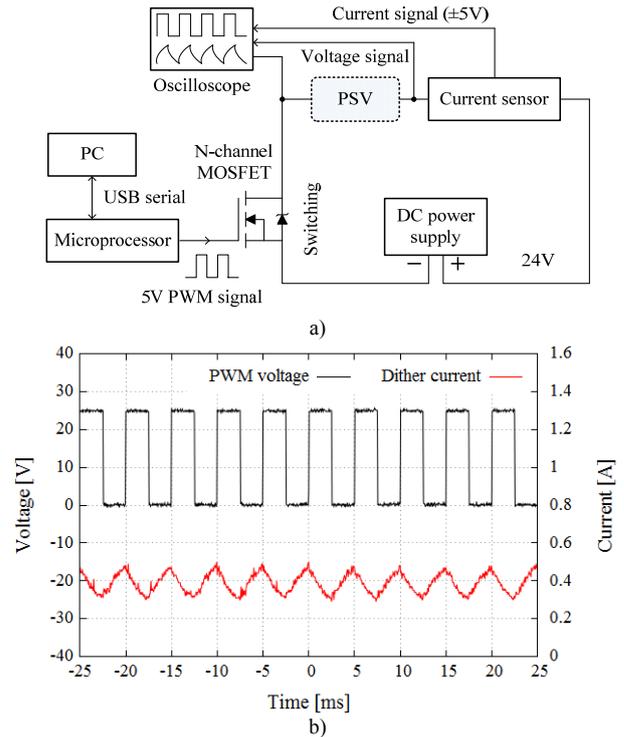


Figure 5. Experimental set-up and results: a) schematic diagram of the experimental set-up, b) waveform at 200 Hz and 50% duty cycle

In the experiments, the average current and the dither amplitude were measured by varying either the PWM frequency or the duty cycle. As shown in Fig. 6, the measured current data ($I_{avg,m}$ and $I_{amp,m}$) are similar to the calculated currents of the mathematical analysis (I_{avg} and I_{amp}). The maximum difference between the measured and the calculated current (approximately 0.05 A) is seen at the PWM frequency of 100 Hz and the duty cycle of 50%, as shown in Fig. 6(a). Most of the measured average current is less than the calculated average current, whereas the measured dither amplitude is always more than the calculated dither amplitude. These differences in the dither amplitude are caused by factors such as noise in the current measurement and the effect of temperature on coil resistance and inductance. In particular, the noise in the current signal makes it difficult to determine the exact current level, whereupon the average of the fluctuations is ultimately selected as a current level, so the dither amplitude is always larger than the calculated current.

Although there are some the differences between the measured and the calculated currents, the current characteristics observed in the experiments are totally

identical to those of the mathematical analysis. At the fixed duty ratio (50%), the average current is nearly steady and the dither amplitude decreases negative exponentially as the PWM frequency increases. At the fixed PWM frequency (200 Hz), also, the average amplitude is proportional to the duty cycle and the dither amplitude is maximized only at 50% duty cycle. Therefore, the mathematical model can be considered reasonable in spite of some differences.

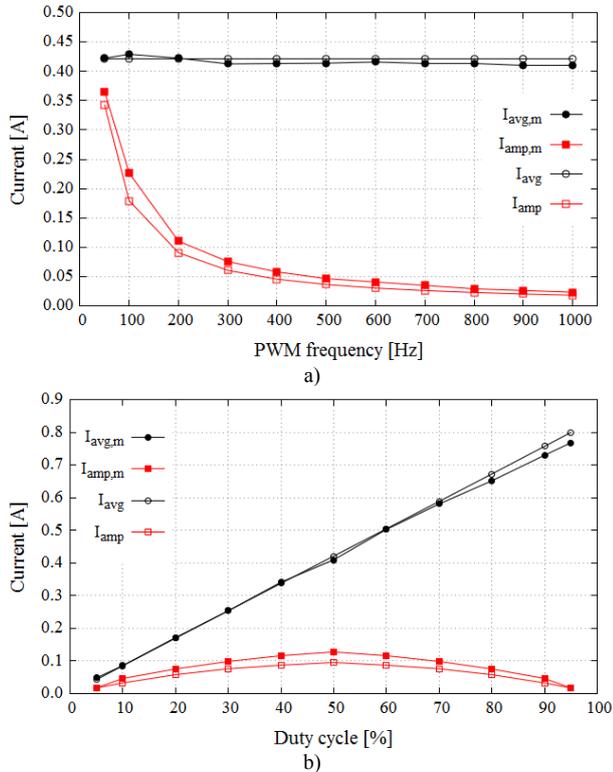


Figure 6. Comparison of the experimental results and mathematical analysis: a) current according to the PWM frequency at 50% duty cycle, b) current according to the duty cycle at 200 Hz

IV. NEW METHOD AND VERIFICATION

A. New Method and Implementation Details

The feasibility of controlling the average current and the dither amplitude independently within a limited range was confirmed from the mathematical analysis and was validated by the experiments. Thus, this paper proposes a new method that varies the PWM frequency and the duty cycle simultaneously in order to obtain the desired average current and constant dither amplitude. The procedure of a new method is explained as follows.

First, the lower and the upper bound of the PWM frequency and duty cycle are defined as shown in Fig. 7(a). Herein, the range of the PWM frequency must be selected carefully because the PSV might not be able to follow very high PWM signals as well as too low ones. Hence, the proper frequency should be determined experimentally; generally it is within the range of 50 Hz to 1000 Hz. Similarly, the duty cycle is commonly within the range of 5% to 95% to avoid the dead zone.

Then, the average current and the dither amplitude according to the PWM frequency are calculated to investigate the controllable current range. Fig. 7(b) shows the calculation result of the controllable current range. The dither amplitudes are diminished drastically as the PWM frequencies increase, and maximized at half of the

maximum average current, i.e. a 50% duty cycle. The area under the curves indicates the controllable current range, which can be achieved by varying the PWM frequency and the duty cycle properly. Therefore, the dither amplitude, which is to be maintained at constant level, must be selected within the feasible area in Fig 7(b). If a dither amplitude less than 0.07 A is chosen, it can be kept constant in all average currents according to Fig. 7(b). The values of the PWM frequency and duty cycle to obtain the desired average current and constant dither amplitude can be calculated by solving the two simultaneous equations, (10) and (11).

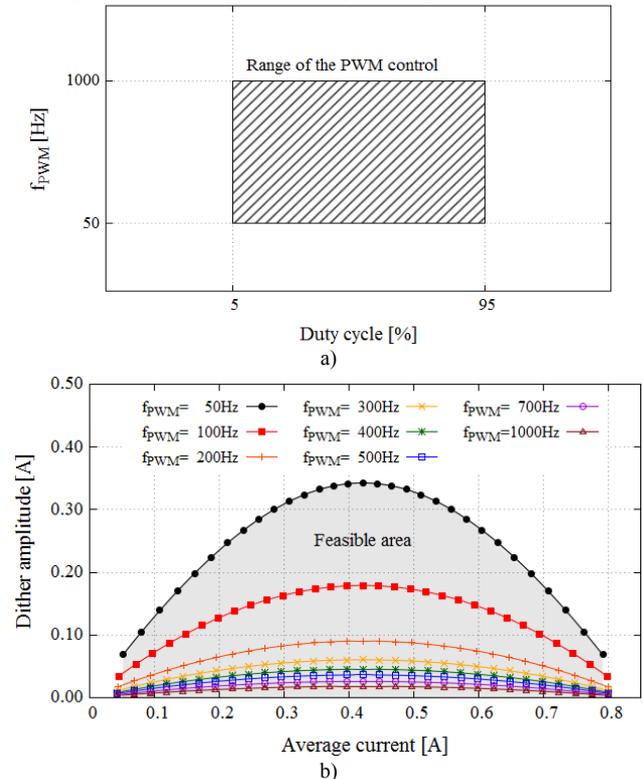


Figure 7. Determination of the controllable current: a) bounds of the PWM frequency and duty cycle, b) controllable current range

If a PC-based controller is used to control a PSV, the exact solution of the two equations can be calculated by mathematical computing software such as MATLAB or Maple. They can be linked to a control program written in LabVIEW or C code to provide exact solutions quickly. If a stand-alone type of controller such as a microprocessor is used, on the other hand, it is difficult to find exact solutions of the two simultaneous equations because they include exponential functions. In this case, instead of exact solutions, approximate solutions can be obtained by simplifying the two simultaneous equations. As stated in Section II, the average current is proportional to the duty cycle and is nearly steady regardless of the PWM frequency. This relationship can be observed obviously by plotting Fig 4 onto the two-dimensional (2-D) plane, as shown in Fig. 8(a). Thus, (10) can be simplified as

$$I_{avg} = k_1 \cdot D \quad (12)$$

That is, (10) can be expressed as a function of the duty cycle only, and therefore the duty cycle can be determined by the desired average current without solving the equations. In like manner, the dither amplitude according to the PWM frequency (Fig. 4) can be plotted as shown in Fig. 8(b). The dither amplitude is influenced by both the duty cycle and the

PWM frequency. However, the PWM frequency can also be determined by the desired dither amplitude only because the duty cycle is already determined from the average current. For convenient calculation, the relationship presented in Fig. 8(b) was modified to Fig. 8(c), and (11) was simplified as

$$\frac{1}{I_{amp}} = k_2 \cdot f_{PWM} \quad (13)$$

Therefore, the PWM frequency can be determined by the reciprocal of the desired dither amplitude. The slopes of each simplified equation (k_1 , k_2) must be determined and programmed into a stand-alone controller in advance to obtain approximate solutions. Although there are some errors in simplifying the equations, approximate solutions in this study are close to the exact solutions with less than 2% relative error. These errors may seem to be too large to obtain proper solutions. However, they are acceptable in the actual control of the PSV because the errors of the average current and the dither amplitude were less than 0.03 A and 0.008 A, respectively.

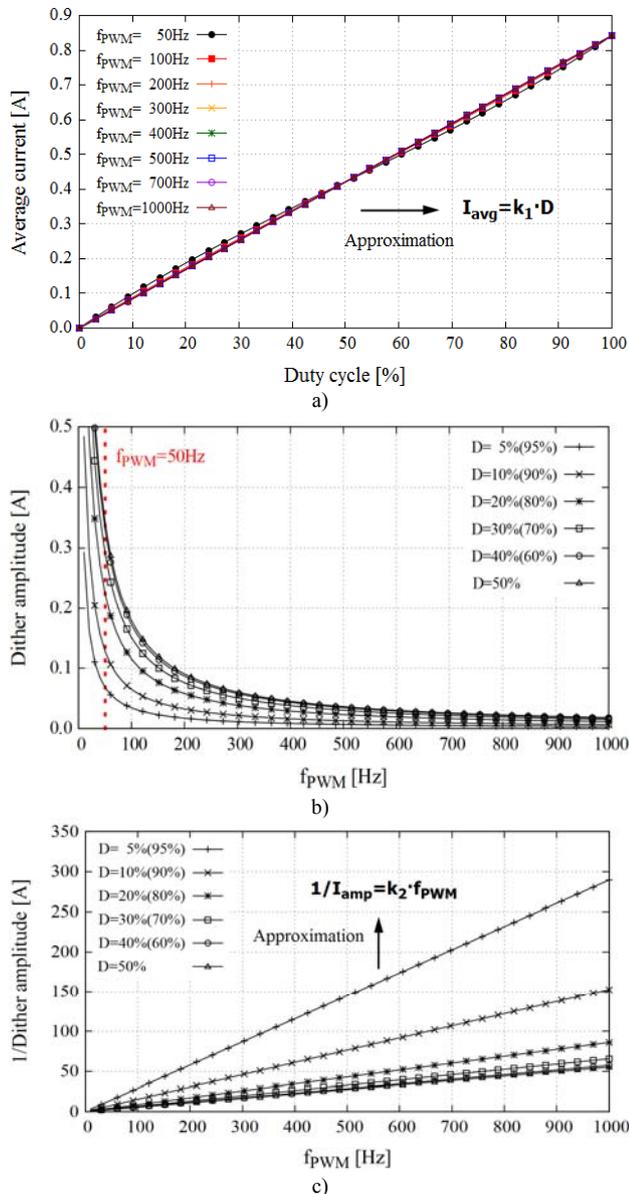


Figure 8. Plots to obtain approximate solutions: a) average current according to duty cycle, b) dither amplitude according to PWM frequency, c) reciprocal of the dither amplitude according to PWM frequency

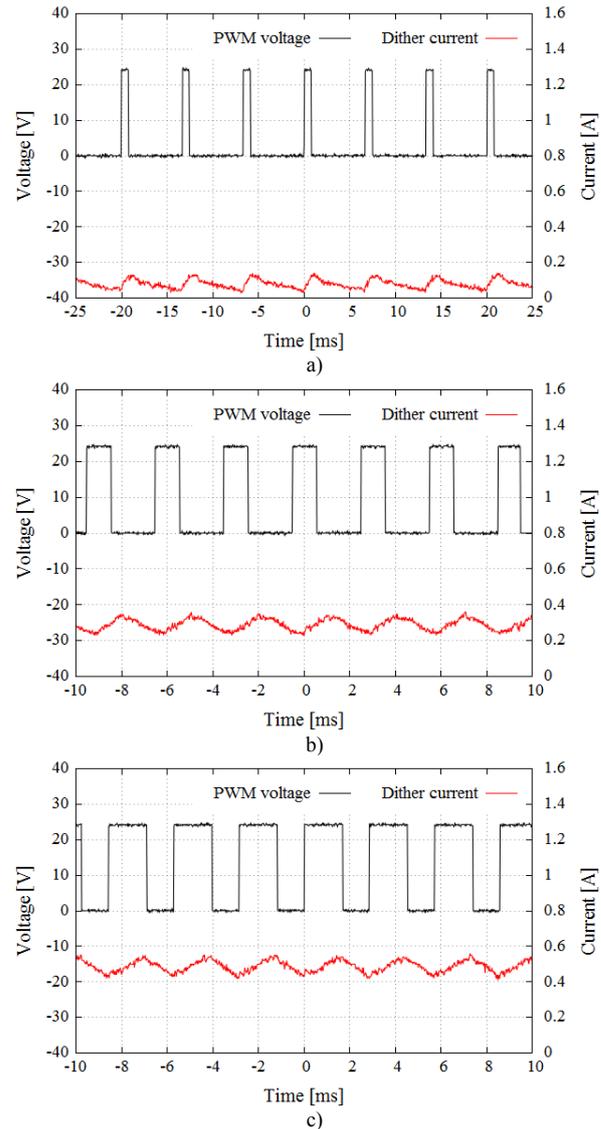
B. Verification of the New Method

To verify the proposed method, an experiment was performed as summarized in Table I. In the experiment, the desired average current was varied within the range of 0.1 A to 0.7 A, and the dither amplitude was maintained at a constant 0.05 A. As shown in Fig. 7, the desired values are within the feasible area. The PWM frequency and the duty cycle for obtaining the desired values were calculated in the microprocessor using the two simplified equations, (12) and (13), and they are listed in Table I.

As shown in Fig. 9, the experimental results show that the desired dither amplitude was 0.05 A, but the measured dither amplitude was kept approximately 0.09 A. These differences correspond to the error between the calculation and the measurement as mentioned in Section III. Even though some error exists, it is obviously demonstrated that the dither amplitude is successfully maintained at a constant level by using the proposed method.

TABLE I. EXPERIMENT FOR VERIFICATION OF THE NEW METHOD

Desired value		Input variable (exact)		Input variable (approximate)	
I_{avg} [A]	I_{amp} [A]	f_{PWM} [Hz]	D [%]	f_{PWM} [Hz]	D [%]
0.1	0.05	149.36	11.66	152.12	11.88
0.3		333.99	35.59	302.46	35.63
0.5		350.36	59.40	349.74	59.38
0.7		202.25	83.27	204.11	83.13



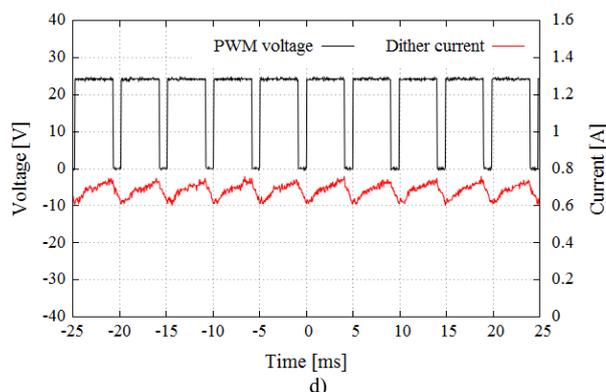


Figure 9. Constant dither amplitude for different average current levels: a) $I_{avg}=0.1$ A, b) $I_{avg}=0.3$ A, c) $I_{avg}=0.5$ A, d) $I_{avg}=0.7$ A

V. CONCLUSION

This paper presents a new method for maintaining dither amplitudes in low frequency PWM. Throughout the mathematical analysis, the effect of the PWM frequency and duty cycle on the average current and the dither amplitude was investigated, and it was presented on a three-dimensional plot as shown in Fig. 4. From the analysis of this plot, the feasibility of controlling the average current and dither amplitude independently within a limited range was confirmed, and the new method was established. This method requires only the calculation for determining a proper PWM frequency and duty cycle, so it is possible to improve the performance of a PSV without additional devices or cost. In addition, it can be easily applied to existing PSV systems by modifying the source code of the main controller, if a digital control system is being used in the target system.

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