A Novel Robust Interacting Multiple Model Algorithm for Maneuvering Target Tracking

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***Abstract*—In this paper, the state estimation problem for discrete-time jump Markov systems is considered. A minimax filtering technique, interacting multiple model algorithm based on game theory, is developed for discrete-time stochastic systems. Filter performance improvement in presence of model uncertainties, measurement noise, and unknown steering command of the maneuvering target is illustrated. It is shown that the technique presented in this paper has a better performance in comparison with the traditional Kalman filter with minimum estimation error criterion for the case of worst possible steering command of target. In particular, simulation results illustrate the improved performance of the proposed filter compared to Interacting Multiple Model (IMM), diagonal-matrix-weight IMM (DIMM), and IMM based on H_∞ (IMMH) filters.**

***Index Terms*—markov processes, infrared sensor, radar, state estimation, filtering algorithm.**

I. INTRODUCTION

Jump Markov System (JMS) is an important topic in many fields such as target tracking [1], networked control systems [2], process monitoring and fault detection [3], filter design, and signal processing [4]. State estimate problem of JMS is an important issue in the maneuvering target tracking systems. The problem of target tracking is a Bayesian filtering problem to estimate characteristics of the target such as position, velocity, and acceleration in 3D Cartesian coordinates. JMS is often utilized for modeling the motion of a maneuvering target that can switch between a finite number of dynamics. The recursive optimal filter for state estimation problem of JMS has been developed in [5-6]. The approach in that paper is rather weak in the presence of unknown measurement noise. Therefore, the state estimation problem of JMS for target tracking systems in the presence of unknown noise and steering command is an interesting topic to pursue.

A promising approach to track a maneuvering target is the interacting multiple model (IMM) algorithm. One of the important features of IMM is that the state estimates of the target and the covariance matrices from multiple models are fused according to a Markov model [7-8]. The IMM algorithm has been applied to track a three-dimensional maneuvering target with passive infrared sensors in [9]. In [10], a converted measurement IMM filter is presented for tracking a maneuvering target using Radar/IR heterogeneous sensors. By analyzing the influence of rate measurement, a new distributed fusion method of radar/IR tracking system based on separation and combination of the measurements is proposed in [11]. To overcome the weakness of IMM algorithm in the presence of unknown measurement noise and steering command of the target, Li has combined the IMM approach with H_∞ technique in order to obtain a robust filter [12].

Measurement noise, unknown steering command, and model uncertainties of the target are some of the challenging problems in target tracking systems [13-17]. To overcome these challenges, for bounded signals, minimax algorithms are used by some researchers to track maneuverable targets [18-19]. In contrast to Kalman filter which requires an accurate system and measurement noise models, minimax algorithms try to minimize the effect of the estimation errors under worst possible noise sequence without requiring accurate system and measurement noise models. Therefore, minimax algorithms seem to be more applicable in real world applications. In [18-19], the minimax algorithm is utilized to minimize estimation error by assuming worst case conditions for system and measurement noise sequences. In [20], the target tracking problem is formulated as zero-sum game and a minimax algorithm is developed to estimate target position in sensor networks. In [21], an optimal minimax filter is presented for tracking a target based on the relationship between the measurements and states. In [22], the problem of H_∞ optimal state estimation filter of linear continuous-time system is evaluated when target tracking problem is formulated as a stochastic game. A reduced minimax state estimation filter is proposed in [23] and a version of minimax state estimation filter is developed for singular linear stationary continues-time dynamic systems in [24]. A discreet-time version of the state estimation filter used in [22] has been developed in [25]. In [22],[25], no constrains are assumed for the state variables. The constraints on the state variables are commonly neglected in state estimation filter design procedure. To extend minimax filter for constrained state variables of linear discrete-time dynamic systems, a constrained state estimation filter is developed in [26].

In this paper, the robust state estimation problem is investigated for a class of JMSs. In [27], our previous work, a minimax filter based on game theory is proposed for tracking a maneuvering target. The minimax filtering is introduced into IMM algorithm for maneuvering target tracking under worst possible steering command of the target. The proposed minimax filter is a robust filter that minimizes the estimation error by considering the worst possible steering command [26-27]. The key difference between the proposed and the existing IMM filters is that in the proposed approach a game theoretic formulation is developed to solve the target tracking problem. Minimax filtering is chosen to deal with the state estimates problem in

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view of the following advantages of minimax estimate [26]: (1) minimax filtering provides a rigorous method for dealing with systems that have model uncertainty; (2) minimax filtering can be utilized to minimize the estimate error under worst possible noise sequence; and (3) minimax filtering is more feasible for JMSs with model uncertainties and unknown measurement noise.

The paper is organized as follows. In Section II, the problem of state estimation of discrete-time JMS is presented. In Section III, an IMM version of the minimax filter is developed. Simulation results for different scenarios and comparisons with IMM, IMMH [12], and DIMM [6] filters are illustrated in Section IV. The conclusion is presented in Section V.

II. Problem Formulation

Consider the JMS given by (1) where matrix functions $\mathbf{A}_j, \mathbf{B}_j$ and $f(\bar{x}_k)$ are known with compatible dimensions [6]. Also , k is the scan time index, $j \in S = \{1,2,...,s\}$ denotes the model, $x_k \in R^n$ is state vector, $y \in R^p$ is measurement vector, $v_k^j \in R^q$ is process noise vector, and $\omega_k^j \in R^p$ is measurement noise vector.

$$\begin{cases} \bar{x}_{k+1} = \mathbf{A}_j \bar{x}_k + \mathbf{B}_j v_k^j \\ y_k = f(\bar{x}_k) + \omega_k^j \end{cases} \tag{1}$$

It is assumed that v_k^j and ω_k^j are white noise processes that are mutually uncorrelated and satisfy (2) and (3) for each i, respectively. In (2) and (3), $E(\bullet)$ is the expected value and $\delta_{k(k-i)}$ is Kronecker delta function. Without loss of generality, adversary inputs, $\widetilde{B}\mu_k$, can be considered in (1), but for convenience, they are eliminated in this section.

$$E\left(v_k^j\right) = 0, E\left(v_k^j \left[v_{k-i}^j\right]^T\right) = \boldsymbol{R}^j \delta_{k(k-i)},\ j \in S \tag{2}$$

$$E\left(\omega_k^j\right) = 0, E\left(\omega_k^j \left[\omega_{k-i}^j\right]^T\right) = \boldsymbol{M}^j \delta_{k(k-i)},\ j \in S \tag{3}$$

System dynamics are modeled as a finite Markov chain with known model transitions probabilities from model i at time $k-1$ to model j at time k as (4) and (5) [7]. In (4), $\widetilde{M}_k^j$ is the flight model j at scan time index k.

$$\pi_{ij} = \Pr ob\left\{\widetilde{M}_k^j \middle| \widetilde{M}_{k-1}^i\right\} = \boldsymbol{P}\left\{\widetilde{M}_k^j \middle| \widetilde{M}_{k-1}^i\right\} \tag{4}$$

$$0 \le \pi_{ij} \le 1, \sum_{i=1}^{s} \pi_{ij} = 1,\ i, j \in S \tag{5}$$

The initial state distribution of Markov chain is $\varphi = [\varphi_1, ..., \varphi_s]$ as (6).

$$0 \le \varphi \le 1, \sum_{j=1}^{s} \varphi_j = 1,\ j \in S \tag{6}$$

A Markov chain is utilized to describe the behavior of system dynamics. In [27], a minimax filter based on game theory for tracking a maneuvering target using radar/infrared (IR) sensors is proposed. In there, the tracking problem in target tracking systems is formulated as a zero-sum to obtain the best estimate of position and velocity of target in 3D Cartesian coordinates. The minimax filter based on game theory guarantees minimum estimation error achievement under worst steering command of the target. IMM algorithms are commonly used for state estimation of maneuvering targets. However, IMM algorithms are weak when measurement noise and steering command of the target are unknown. Therefore, in the following, a novel IMM algorithm based on minimax filtering is developed.

III. IMM Algorithm Based on Minimax Filtering Problem

A. Minimax Filter

In this section, minimax filter (MF) is described. Consider the discrete-time dynamic model in (7). In (7), k, $\bar{x}_k$, y_k, and $f(\bar{x}_k)$ are the same as before, $v_k \in R^m$ is process noise vector, $\omega_k \in R^r$ is measurement noise vector, and $\boldsymbol{\mu}_k$ is the adversary steering command vector that is of course unknown to us. A, and B are known matrices, and $\widetilde{B}$ is a matrix with appropriate dimension relating state of the system to steering command of the maneuvering target. Once again, it is assumed that v_k and ω_k are white noise processes that are mutually uncorrelated and satisfy (8) and (9) for each i, respectively. In general, $\boldsymbol{\mu}_k$ is selected such that the worst case scenario for state estimation is obtained. According to [27], this worst case is achieved when $\boldsymbol{\mu}_k$ is given by (10).

$$\begin{cases} \bar{x}_{k+1} = A\bar{x}_k + Bv_k + \widetilde{B}\boldsymbol{\mu}_k \\ y_k = f(\bar{x}_k) + \omega_k \end{cases} \tag{7}$$

$$E(v_k) = 0, E\left(v_k [v_{k-i}]^T\right) = R\delta_{k(k-i)} \tag{8}$$

$$E(\omega_k) = 0, E\left(\omega_k [\omega_{k-i}]^T\right) = M\delta_{k(k-i)} \tag{9}$$

$$\mu_k = \widetilde{K}_a^{MF}\left(G_k\left(\bar{x}_k - \hat{\bar{x}}_k\right) + n_k\right) \tag{10}$$

In (10), $\widetilde{K}_a^{MF}$ is adversary gain matrix that is calculated at each step, $\hat{\bar{x}}_k$ is an estimate of $\bar{x}_k$, G_k is a measurement gradient matrix, and n_k is white noise process that satisfies (11). In this paper, it is assumed that v_k, ω_k, n_k, and initial condition $\bar{x}_0$ are mutually uncorrelated.

$$E(n_k) = 0, E\left(n_k n_{k-i}^T\right) = N\delta_{k(k-i)} \tag{11}$$

Since in target tracking systems the unbiased state estimation filters are more suitable compared to biased state estimation filters, the filter structure is assumed as (12) with zero mean initial condition where K_f^{MF} is filter gain matrix and $\hat{\bar{y}}_k$ is given by (13). In addition to mathematical tractability of an unbiased filter, the mean value of an unbiased filter equals the true value of the quantity that must be estimated.

$$\hat{\bar{x}}_{k+1} = A\hat{\bar{x}}_k + K_f^{MF}\left(y_k - \hat{\bar{y}}_k\right) \tag{12}$$

$$\hat{\bar{y}}_k = f\left(\hat{\bar{x}}_k\right) \tag{13}$$

Using (12), (13) and defining the following $K_a^{MF} = \widetilde{B}\widetilde{K}_a^{MF}; G_k = C_k; F_k = A - K_f^{MF} C_k + K_a^{MF} C_k$

one can write the estimation error ($e_k = \bar{x}_k - \hat{\bar{x}}_k$) as:

$$e_{k+1} = F_k e_k + Bv_k + K_a^{MF} n_k - K_f^{MF} \omega_k \tag{14}$$

where C_k is the Jacobian of $f\left(\hat{\bar{x}}_{k-1}\right)$. From the above discussion and using [27], in order to define a zero-sum game, the estimation error given by (14) is decomposed into two parts as (15) and a utility function given by (16) is considered. In (15), e_k^f and e_k^a are given by (17) and (18), respectively.

$$e_k = e_k^f + e_k^a \tag{15}$$

$$J_N = trace\left\{\sum_{k=0}^{N-1} W_k E\left\{\left\|e_k^f\right\| - \gamma_k^2 \left\|e_k^a\right\|\right\}\right\} \tag{16}$$

$$e_{k+1}^f = F_k e_k^f + Bv_k - K_f^{MF} \omega_k \tag{17}$$

$$e_{k+1}^a = F_k e_k^a + K_f^{MF} n_k \tag{18}$$

In (16), the scalar $\gamma_k > 0$ is an adjustable parameter that is necessary when $f\left(\bar{x}_k\right)$ has high nonlinearity, N is the time horizon and W_k is any positive definite weighting matrix.

Finally, the recursive minimax filter proposed in [27] has the following form:

$$\begin{aligned} \hat{\bar{x}}_k &= A\hat{\bar{x}}_{k-1} + K_f^{MF}\left(y_{k-1} - \hat{\bar{y}}_{k-1}\right) \\ P_k^{-1} &= Q_{k-1}^{-1} + C_k^T\left(M^{-1} - \gamma_k^{-2} N^{-1}\right)C_k \\ Q_k &= F_k Q_{k-1} F_k^T + BRB^T \\ &\quad + K_f^{MF} M\left(K_f^{MF}\right)^T - \gamma_k^2 K_a^{MF} N\left(K_a^{MF}\right)^T \\ K_f^{MF} &= AP_k C_k^T M^{-1} \\ K_a^{MF} &= \gamma_k^{-2} AP_k C_k^T N^{-1} \end{aligned} \tag{19}$$

where Q_k is state covariance matrix of $\hat{\bar{x}}_k$. Also, the γ_k parameter is chosen such that the trace of the covariance matrix is as small as possible at every step.

Remark1. The minimax filter based on game theory guarantees minimum estimation error under worst steering command of the target. However, the filter is weak when the turn rate, the constant acceleration, and the start and the end time of every maneuver motion are unknown.

B. The Proposed Algorithm

To obtain state estimates of the maneuvering target, the minimax filtering is fused with IMM filter in the proposed algorithm using the following steps:

Step 1: Calculate the mixed initial weight for the filter matched to model $\widetilde{M}_k^j$ ($j \in S$).

$$\begin{aligned} \mu_{i|j}(k \mid k) &= \boldsymbol{P}\left\{\widetilde{M}_{k-1}^i \middle| \widetilde{M}_k^j, y_{k-1}\right\} \\ &= \frac{\boldsymbol{P}\left\{\widetilde{M}_k^j \middle| \widetilde{M}_{k-1}^i, y_{k-1}\right\}\boldsymbol{P}\left\{\widetilde{M}_{k-1}^i \middle| y_{k-1}\right\}}{\boldsymbol{P}\left\{M_k^j \middle| y_{k-1}\right\}} \\ &= \frac{\pi_{ij}\mu_{k-1}^i}{c_j}, \qquad i, j \in S \end{aligned} \tag{20}$$

where c_j is a normalization factor.

Step 2: Calculate the mixed initial state and corresponding covariance for the filter matched to model $\widetilde{M}_k^j$ ($j \in S$).

$$\hat{\bar{x}}_{k|k}^{j0} = \sum_{i=1}^{s} \mu_{i|j}(k \mid k)\hat{\bar{x}}_{k-1}^i \tag{21}$$

$$\begin{aligned} &Q_{k|k}^{j0} \\ &= \sum_{i=1}^{s} \mu_{i|j}(k \mid k)\left\{Q_{k-1}^i \times \left[\hat{\bar{x}}_{k-1}^i - \hat{\bar{x}}_{k|k}^{j0}\right]\left[\hat{\bar{x}}_{k-1}^i - \hat{\bar{x}}_{k|k}^{j0}\right]^T\right\} \end{aligned} \tag{22}$$

In (21) and (22), $\hat{\bar{x}}_{k-1}^i$ is the estimation of the state based on *ith* minimax filter at time $k-1$, and the corresponding covariance is Q_{k-1}^i.

Step 3: Filtering ($j \in S$).

$$\begin{aligned} \hat{\bar{x}}_k^j &= \mathbf{A}_j \hat{\bar{x}}_{k|k}^{j0} + K_f^j\left(y_{k-1} - \hat{\bar{y}}_{k-1}^{j0}\right) \\ Q_k^j &= F_k^j Q_k^{j0} F_k^{j^T} + \mathbf{B}_j R \mathbf{B}_j^T \\ &\quad + K_f^j M\left(K_f^j\right)^T - \gamma_k^2 K_a^j N\left(K_a^j\right)^T \\ K_f^j &= \mathbf{A}_j P_k C_k^{j^T} \boldsymbol{M}^{-1} \\ K_a^j &= \gamma_k^{-2} \mathbf{A}_j P_k C_k^{j^T} \boldsymbol{N}^{-1} \end{aligned} \tag{23}$$

where $F_k^j = \mathbf{A}_j - K_f^j C_k^j + K_a^j C_k^j$, $\hat{\bar{y}}_{k-1}^{j0} = f\left(\hat{\bar{x}}_{k|k}^{j0}\right)$, and C_k^j is the Jacobian of $f\left(\hat{\bar{x}}_{k|k}^{j0}\right)$.

Step 4: Update the mode probability

$$\Lambda_k^i = \widetilde{\boldsymbol{N}}\left(\widetilde{v}_k^i, 0, S_k^i\right) \tag{24}$$

$$\mu_k^i = \boldsymbol{P}\left\{M_k^i \middle| y_k\right\} = \frac{1}{\bar{c}}\Lambda_k^i c_i \tag{25}$$

where $\bar{c}$ is a normalization factor, and $\widetilde{\boldsymbol{N}}\left(\mathrm{X}, \overline{\mathrm{X}}, \mathbf{Q}\right)$ denotes a normal probability distribution function of X with mean $\overline{\mathrm{X}}$ and covariance $\mathbf{Q}$. In (24), $\widetilde{v}_k^i$ and S_k^i are as (26) and (27), respectively.

$$\widetilde{v}_k^i = y_k - f\left(\hat{\bar{x}}_k^i\right) \tag{26}$$

$$S_k^i = C_k^i Q_k^i C_k^{i^T} + \boldsymbol{M} \tag{27}$$

Step 5: Estimate fusion

$$\hat{\bar{x}}_k = \sum_{i=1}^{s} \mu_k^i \hat{\bar{x}}_k^i \tag{28}$$

$$Q_k = \sum_{i=1}^{s} \mu_k^i\left\{Q_k^i + \left[\hat{\bar{x}}_k^i - \hat{\bar{x}}_k\right]\left[\hat{\bar{x}}_k^i - \hat{\bar{x}}_k\right]^T\right\} \tag{29}$$

Remark2. The main differences between the proposed minimax state estimation algorithm and the IMM algorithm are twofold. First, the tracking problem in target tracking systems is formulated as a zero-sum game. The advantage of this formulation is that the effect of the estimation errors is minimized under worst possible noise sequence. Second, instead of Kalman filters, a group of minimax filters are combined with the IMM. The advantage of this approach is that filtering technique can improve filter performance in the presence of model uncertainties, measurement noise, and unknown steering command of the maneuvering target.

Remark3. Although the proposed algorithm has a computational cost that is slightly higher than IMM due to more computational burden, it provides a more accurate and stable estimate. The proposed algorithm has a filter structure similar to that of the IMMH algorithm, and it has an analogous computational complexity. Compared to DIMM algorithm, the proposed algorithm has less computational complexity.

IV. SIMULATION RESULTS

To illustrate the performance of the proposed filter, different scenarios are considered, and the simulation results are compared with IMM, IMMH [12] and DIMM [6] filters. The proposed filter is denoted by IMM-MF in figures displaying the simulations results. We consider three tracking scenarios with varying measurement noise and evaluate the performance of proposed algorithm for these scenarios. Simulations are performed using Matlab software.

A. Simulation Setup

In this section, combination of radar and IR sensors has been considered for tracking a maneuvering target. The sampling rate is *T=0.5* second. It is assumed that radar and IR sensors lie in the same platform. Therefore, it can be assumed that the two sensors are located in the same position. By considering a maneuverable target in 3D Cartesian coordinates, the measuring geometry relationship between target and sensors platform is shown in Fig. 1.

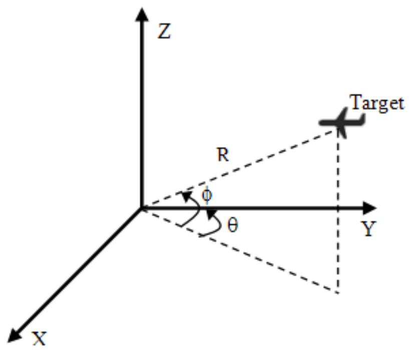

Figure 1. measuring geometry relationship between target and sensors platform

The measurement model for the 2D radar sensor is as (30) where R_k^R is the range and θ_k^R is the azimuth angle given by (31) and (32), respectively, and ω_k^{Radar} is the measurement noise vector for the radar sensor.

$$z_k^R = h_{Radar}(\bar{x}_k) = \begin{bmatrix} R_k^R \\ \theta_k^R \end{bmatrix} + \omega_k^{Radar} \tag{30}$$

$$R_k^R = \sqrt{x_k^2 + y_k^2 + z_k^2} \tag{31}$$

$$\theta_k^R = \tan^{-1}\left(y_k / x_k \right) \tag{32}$$

The measurement model for the IR sensor is as (33) where θ_k^I is the azimuth angle and ϕ_k^I is the elevation angle given by (34) and (35), respectively, and ω_k^{IR} is measurement noise vector for the IR sensor.

$$z_k^{IR} = h_{IR}(\bar{x}_k) = \begin{bmatrix} \theta_k^I \\ \phi_k^I \end{bmatrix} + \omega_k^{IR} \tag{33}$$

$$\theta_k^I = \tan^{-1}\left(y_k / x_k \right) \tag{34}$$

$$\phi_k^I = \tan^{-1}\left(z_k \Big/ \sqrt{x_k^{\,2} + y_k^{\,2}} \right) \tag{35}$$

Considering all the above, $f(\bar{x}_k)$, measurement model, can be written as:

$$f(\bar{x}_k) = \begin{bmatrix} z_k^R \\ z_k^{IR} \end{bmatrix} = \begin{bmatrix} R_k^R & \theta_k^R & \theta_k^I & \phi_k^I \end{bmatrix}^T + \begin{bmatrix} \omega_k^{Radar} \\ \omega_k^{IR} \end{bmatrix} \tag{36}$$

The normal measurement noise vector for the radar sensor is a zero mean Gaussian sequence with standard deviation of 15 m for range and 5 milliradian (mrad) for azimuth angle. The normal measurement noise vector for IR sensor is a zero mean Gaussian sequence with standard deviation of 4 mrad for both azimuth and elevation angles. In the simulation, the covariance matrix, n_k , is as (37).

$$N = \begin{bmatrix} 30^2 & 0 & 0 & 0 \\ 0 & 2.5\times10^{-4} & 0 & 0 \\ 0 & 0 & 4\times10^{-4} & 0 \\ 0 & 0 & 0 & 4\times10^{-4} \end{bmatrix} \tag{37}$$

Root Mean Square Error (RMSE) and Cumulative Root Mean Square Error (CRMSE) are frequently used as performance indices in target tracking systems. RMSE represents the standard deviation of the differences between estimated and actual values. The RMSE of position and velocity estimates, given by (38) and (39), respectively, are used in this paper to illustrate the performance of the proposed filter. The CRMSE of position and velocity estimates are given by (40) and (41), respectively. In each scenario, the averaged results based on 100 Monte-Carlo iterations are illustrated.

$$RMSE_P(k) = \sqrt{\frac{1}{3}\left((x_k - \hat{x}_k)^2 + (y_k - \hat{y}_k)^2 + (z_k - \hat{z}_k)^2 \right)} \tag{38}$$

$$RMSE_V(k) = \sqrt{\frac{1}{3}\left(\left(\dot{x}_k - \dot{\hat{x}}_k\right)^2 + \left(\dot{y}_k - \dot{\hat{y}}_k\right)^2 + \left(\dot{z}_k - \dot{\hat{z}}_k\right)^2 \right)} \tag{39}$$

$$CRMSE_P(K) = \frac{1}{KT}\sum_{k=1}^{K}\sqrt{\frac{k}{3K}\left((x_k - \hat{x}_k)^2 + (y_k - \hat{y}_k)^2 + (z_k - \hat{z}_k)^2 \right)} \tag{40}$$

$$CRMSE_V(K) = \frac{1}{KT}\sum_{k=1}^{K}\sqrt{\frac{k}{3K}\left(\left(\dot{x}_k - \dot{\hat{x}}_k\right)^2 + \left(\dot{y}_k - \dot{\hat{y}}_k\right)^2 + \left(\dot{z}_k - \dot{\hat{z}}_k\right)^2 \right)} \tag{41}$$

For tracking of target, discrete-time constant acceleration (CA) and constant velocity (CV) dynamic models are employed with different process noises. These models are surveyed in [28]. The discrete-time CV and CA dynamic models are given by (42) and (43), respectively.

$$\bar{x}_k = [x_k\ \dot{x}_k\ y_k\ \dot{y}_k\ z_k\ \dot{z}_k]^T$$

$$\mathbf{A}_1 = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & A_1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{42}$$

$$\mathbf{B}_1 = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_1 & 0 \\ 0 & 0 & B_1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}$$

$$\bar{x}_k = [x_k\ \dot{x}_k\ \ddot{x}_k\ y_k\ \dot{y}_k\ \ddot{y}_k\ z_k\ \dot{z}_k\ \ddot{z}_k]^T$$

$$\mathbf{A}_2 = \begin{bmatrix} A_2 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \tag{43}$$

$$\mathbf{B}_2 = \begin{bmatrix} B_2 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_2 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} T^2/2 \\ T \\ 1 \end{bmatrix}$$

The process noise covariance matrices for CV and CA models are given by (44) and (45), respectively. In (44) and (45), $r_1^1 = r_2^1 = 4^2$, $r_3^1 = 1.5^2$, $r_1^2 = r_2^2 = 2^2$, and $r_3^2 = 1.5^2$. The measurement noise covariance matrix is chosen as (46) for $i = 1,2$. In (46), $\sigma_R^R = 15^2$, $\sigma_\theta^R = 2.5\times10^{-5}$, $\sigma_\theta^{IR} = 1.6\times10^{-5}$, and $\sigma_\phi^{IR} = 1.6\times10^{-5}$.

$$R^1 = \begin{bmatrix} r_1^1 & 0 & 0 \\ 0 & r_2^1 & 0 \\ 0 & 0 & r_3^1 \end{bmatrix} \tag{44}$$

$$R^1 = \begin{bmatrix} r_1^2 & 0 & 0 \\ 0 & r_2^2 & 0 \\ 0 & 0 & r_3^2 \end{bmatrix} \tag{45}$$

$$M^i = \begin{bmatrix} \sigma_R^R & 0 & 0 & 0 \\ 0 & \sigma_\theta^R & 0 & 0 \\ 0 & 0 & \sigma_\theta^{IR} & 0 \\ 0 & 0 & 0 & \sigma_\phi^{IR} \end{bmatrix} \tag{46}$$

The initial error covariance and mode transition matrix are assumed as (47) and (48), respectively.

$$Q_0 = diag([111111111]) \tag{47}$$

$$[\pi_{ij}] = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix} \tag{48}$$

B. Scenario I

In first scenario, simulations are performed with nominal noise statistics of radar/IR sensors. The target is assumed to have the following constraints for Scenario I.

- Target position at t=0: [400, 800, 12500] in meters.
- Target velocity at t=0: [80, 70, -80] m/s.
- Constant velocity, $0 < t \le 5$
- Constant acceleration of [15, -9, -9] m/s^2, $5 < t \le 10$
- Constant acceleration of [0, 6, 0] m/s^2, $10 < t \le 20$
- Constant acceleration of [-15, 6, 0] m/s^2, $20 < t \le 30$
- Constant acceleration of [-3, 3, -9] m/s^2, $30 < t \le 37$
- Constant velocity, $37 < t \le 50$

The initial state of the tracking algorithms $\hat{x}_0$ can be calculated by utilizing two-step extrapolation algorithm and coordinate transform of the measurements. The actual trajectory is shown in Fig. 2. Fig. 3, and Fig. 4 display RMSE of position, and velocity for IMM-MF, IMM, IMMH, and DIMM filters, respectively. The results show that all filters can estimate the position and velocity of maneuverable target in 3D Cartesian coordinates with reasonable accuracy. The IMM-MF results in average position error of around 39.59 m, while the IMM, IMMH and DIMM filters give average position errors of around 125.29 m, 48.30 m, and 72.26 m, respectively. The IMM-MF average velocity error is around 19.47 m/s, while the IMM, IMMH, and DIMM average velocity errors are around 57.68 m/s, 31.39 m/s, and 31.61 m/s, respectively.

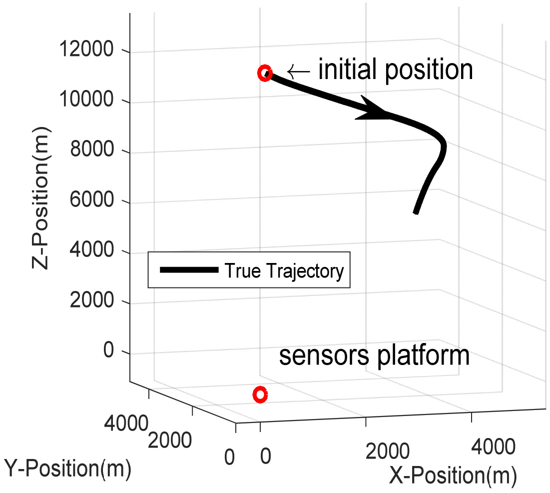

Figure 2. Target trajectory in scenario I

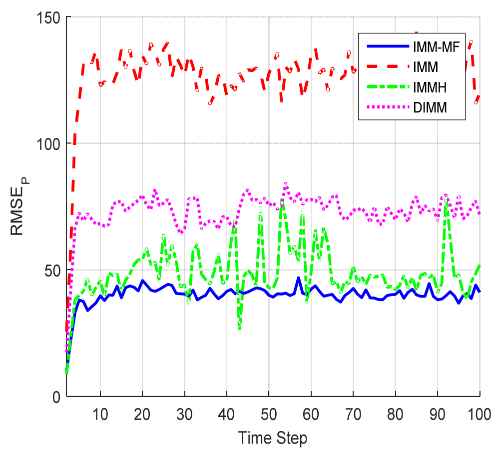

Figure 3. RMSE of position in scenario I

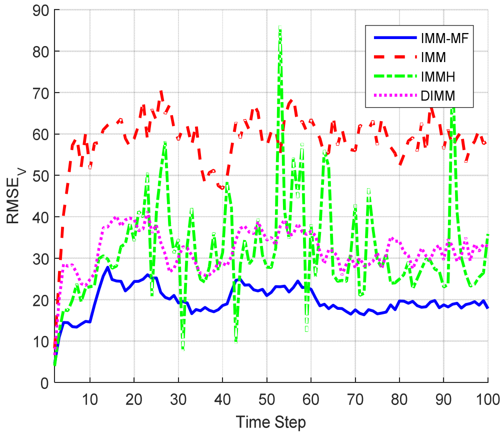

Figure 4. RMSE of velocity in scenario I

CRMSE for different filters are shown in Table I. It can be seen that the $CRMSE_p$ of IMM-MF is reduced by 43% for IMM filter, 9% for IMMH filter, and 26% for DIMM filter. The $CRMSE_v$ of IMM-MF is also reduced by 36% for IMM filter, 20% for IMMH filter, and 21% for DIMM filter.

TABLE I. CRMSE COMPARISON OF DIFFERENT FILTERS FOR SCENARIO I

Filter	Position (*m*)	Velocity (*m/s*)
IMM-MF	4.9196	3.4295
IMM	8.7516	5.39441
IMMH	5.4224	4.3328
DIMM	6.6502	4.3845

C. Scenario II

Scenario II is identical to Scenario I except the measurement noise is not nominal (off-nominal noise statistics). To compare the performance of different filters in the presence of an unknown noise with Scenario I, parameters of filters are kept unchanged and the target trajectory is kept the same as Scenario I. Simulation results are shown in Fig. 5 and Fig. 6 when measurement noise vector for the radar sensor is a zero mean Gaussian sequence with standard deviation of 50 m for range and 10mrad for azimuth angle, and measurement noise vector for IR sensor is a zero mean Gaussian sequence with standard deviation of 12mrad for both azimuth and elevation angles. Fig. 5 and Fig. 6 display RMSE of position and velocity for IMM-MF, IMM, IMMH, and DIMM, respectively. Simulation results show that IMM-MF outperforms IMM, IMMH, and DIMM filters.

The IMM-MF results in average position error of around 163.39 m, while the IMM, IMMH and DIMM filters give average position errors of around 595.57m, 235.38m, and 330.68 m, respectively. The IMM-MF average velocity error is around 83.49 m/s, while the IMM, IMMH, and DIMM average velocity errors are around 667.19 m/s, 227.85 m/s, and 228.14 m/s, respectively.

CRMSE for different filters are shown in Table II. It can be seen that the $CRMSE_p$ of IMM-MF is reduced by 47% for IMM filter, 16% for IMMH filter, and 29% for DIMM filter. The $CRMSE_v$ of IMM-MF is also reduced by 63% for IMM filter, 45% for IMMH filter, and 39% for DIMM filter.

It can be seen that IMM-MF has a higher accuracy estimate than IMMH filter, IMMH filter has a higher accuracy estimate than DIMM algorithm, and DIMM filter has a higher accuracy estimate than IMM algorithm. This is due to the fact that the IMM-MF fused with the minimax filter is more robust than IMM, IMMH, and DIMM filters. Therefore, one can infer that IMM-MF, IMM, IMMH, and DIMM filters can estimate the position and velocity of maneuverable target in 3D Cartesian coordinates with only a reasonable accuracy when the noise statistics are nominal. However, if the noise statistics are not known, then the IMM-MF will perform better than the IMM, IMMH, and DIMM filters.

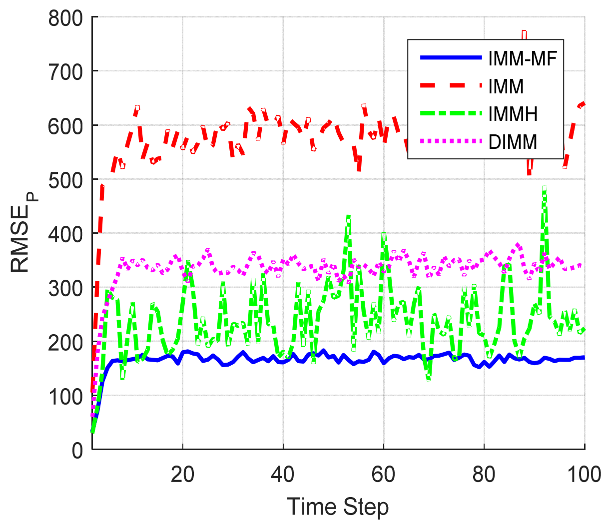

Figure 5. RMSE of position in scenario II

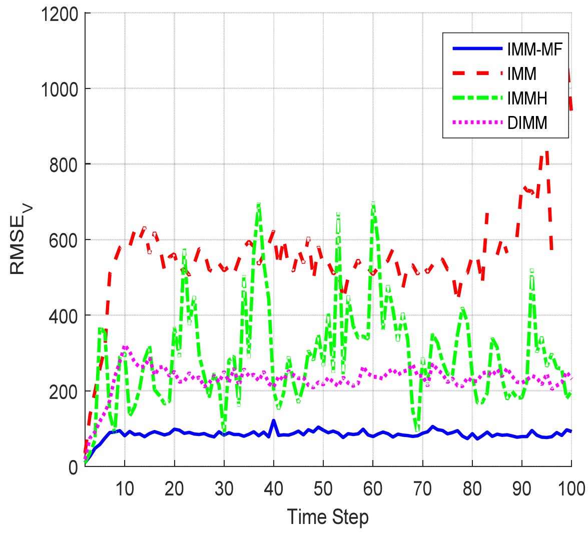

Figure 6. RMSE of velocity in scenario II

TABLE II. CRMSE COMPARISON OF DIFFERENT FILTERS FOR SCENARIO II

Filter	Position (*m*)	Velocity (*m/s*)
IMM-MF	10.01	7.15
IMM	19.06	19.71
IMMH	11.97	13.08
DIMM	14.25	11.82

D. Scenario III

In scenario III, a sophisticated trajectory of the target is designed to evaluate IMM-MF algorithm. Simulations are performed with nominal noise statistics of radar/IR sensors.

The target is assumed to have the following constraints for Scenario III.

- Target position at t=0: [950, 600, 12500] in meters.
- Target velocity at t=0: [90, 40, -140] m/s.
- In horizontal plane (x-y plane):
  - constant turn rate of 0.3 *rad/s* , $0 < t \leq 10$

- constant velocity, $10 < t \leq 20$
- constant turn rate of 0.4 *rad/s*, $20 < t \leq 45$
- constant velocity, $45 < t \leq 50$

- In vertical axis (z-axis):
  - constant velocity, $0 < t \leq 10$
  - constant acceleration of 15 m/s^2, $10 < t \leq 50$

The trajectory for this scenario is shown in Fig. 7.

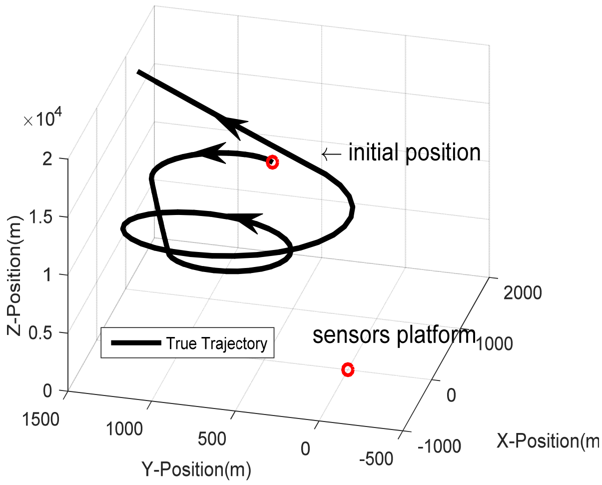

Figure 7. Target trajectory in scenario III

Fig. 8, and Fig. 9 display RMSE of position, and velocity for IMM-MF, IMM, IMMH, and DIMM filters, respectively. It can be seen that the IMM-MF has a higher accuracy than IMM, DIMM, and IMMH filters most of the time. Also, the RMSE of target position of the IMM-MF and IMMH filters is almost the same when the target moves at constant velocity.

The IMM-MF results in average position error of around 56.17 m, while the IMM, IMMH and DIMM filters give average position errors of around 143.95 m, 82.83 m, and 98.98m, respectively. The IMM-MF average velocity error is around 33.51 m/s, while the IMM, IMMH, and DIMM average velocity errors are around 96.01 m/s, 36.25 m/s, and 59.85 m/s, respectively.

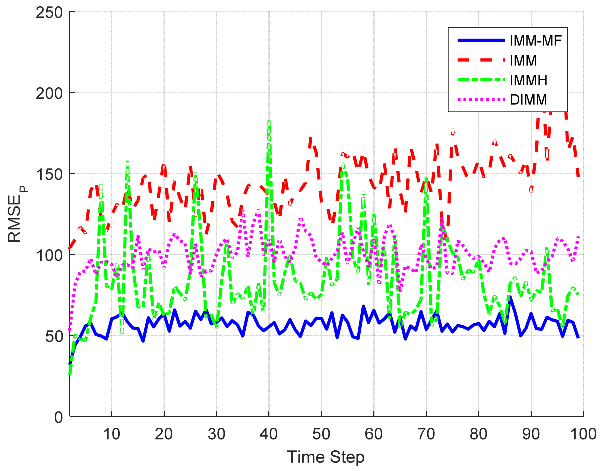

Figure 8. RMSE of position in scenario III

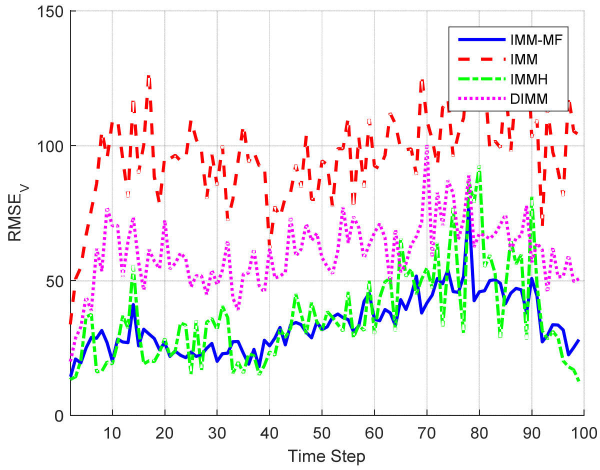

Figure 9. RMSE of velocity in scenario III

V. Conclusion

In this paper, an interacting multiple model algorithm based on minimax filtering was developed for JMSs. A minimax filter based on game theory was developed in our previous paper. The minimax filter guarantees minimum estimation error under worst steering command of the target. Therefore, to minimize the estimation error under the worst case steering command of the target, the minimax filtering was fused with IMM filter. Simulations were performed for different scenarios and the results validated the tracking accuracy advantage of the proposed filter in comparison with IMM, IMMH and DIMM. Research on how to deal with target tracking with asynchronous measurements is left as a future work.

References

[1] C.J. Lee, J.M. Pak, C.K. Ahn, Min, K.M., P. Shi, M.T. Lim, "Multi-Target FIR Tracking Algorithm for Markov Jump Linear Systems Based on True-Target Decision-Making," Neurocomputing, vol. 168, no. 11, pp. 298-307, 2015. doi:10.1016/j.neucom.2015.05.096

[2] Q. Zhang, L. Li, X.G. Yan, S.K. Spurgeon, "Sliding Mode Control for Singular Stochastic Markovian Jump Systems with Uncertainties," Automatica, vol. 79, no. 5, pp. 27-34, 2017. doi:10.1016/j.automatica.2017.01.002

[3] H. Habibi, I. Howard, R. Habibi, "Bayesian Sensor Fault Detection in a Markov Jump System," Asian Journal of Control, In press, 2017. doi:10.1002/asjc.1458.

[4] X. Li, J. Lam, H. Gao, J. Xiong, "H_∞ and H_2 Filtering for Linear Systems with Uncertain Markov Transitions," Automatica, vol. 67, no. 5, pp. 252-266, 2016. doi:10.1016/j.automatica.2016.01.016

[5] X. Fu, Y. Jia, J. Du, F. Yu, "New Interacting Multiple Model Algorithms for the Tracking of the Manoeuvring Target," IET Control Theory & Applications, vol. 4, no. 10,p. 2184-2194, 2010. doi:10.1049/iet-cta.2009.0583

[6] X. Fu,Y. Shang, H. Yuan, "Improved Diagonal Interacting Multiple Model Algorithm for Manoeuvering Target Tracking Based on H_∞ Filter," IET Control Theory & Applications, vol. 9, no. 12, pp. 1887-1892, 2015. doi:10.1049/iet-cta.2014.0685

[7] W. Zhu, W. Wang, G. Yuan, "An Improved Interacting Multiple Model Filtering Algorithm Based on the Cubature Kalman Filter for Maneuvering Target Tracking," Sensors, vol. 16, no. 6, pp. 805-907, 2016. doi:10.3390/s16060805

[8] S. Vasuhi, V. Vaidehi, "Target Tracking Using Interactive Multiple Model for Wireless Sensor Network," Information Fusion, vol. 27, no. 1, pp. 41-53, 2016. doi:10.1016/j.inffus.2015.05.004

[9] F. Dufour, M. Mariton, "Tracking a 3D Maneuvering Target with Passive Sensors," IEEE Transactions on Aerospace Electronic Systems, vol. 27, no. 4, pp. 725–739, 1991. doi:10.1109/7.85047

[10] J.H. Yin, B.Z. Cui, Y.F. Wang, "A Novel Maneuvering Target Tracking Algorithm for Radar/Infrared Sensors," Chin. J. Electron, vol. 19, no. 4, pp. 752-756, 2010.

[11] Q.C. Wang, W.F. Wang, "Tracking Method Based on Separation and Combination of the Measurements for Radar and IR Fusion System," Journal of Systems Engineering and Electronics, vol. 20, no. 2, pp. 241-246, 2009.

[12] W. Li, Y. Jia, J. Du, J. Zhang, "Robust State Estimation for Jump Markov Linear Systems with Missing Measurements," Journal of the Franklin Institute, vol. 350, no. 6, pp. 1476-87, 2013. doi:10.1016/j.jfranklin.2013.04.002

[13] Y. J. He, M. Li, J. L. Zhang, J. P. Yao, "Infrared Target Tracking via Weighted Correlation Filter," Infrared Physics & Technology, vol. 73, pp. 103-114, 2015. doi:10.1016/j.infrared.2015.09.010

[14] M. Ghazal, A. Doustmohammadi, "Multi-sensor Target Tracking in the Presence of Clutter Using Game Theory," Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, First Published, 2017. doi:10.1177/0954410017717534

[15] Y. Ulker, B. Gunsel, "Multiple Model Target Tracking with Variable Rate Particle Filters," Digital Signal Processing, vol. 22, no. 3, pp. 417-429, 2012. doi:10.1016/ j.dsp.2012.01.003

[16] A. Houles, Y. Bar-Shalom, "Multisensor Tracking of A Maneuvering Target in Clutter," IEEE Transaction on Aerospace Electronic Systems, vol. 25, no. 2, pp. 176-189, 1989. doi:10.1109/ TAES.2005.1541458

[17] B. Mocanu, T. Ruxandra, T. Zaharia, "3D Object Metamorphosis with Pseudo Metameshes," Advances in Electrical and Computer Engineering, vol. 15, no. 1, pp. 115-122, 2015. doi:10.4316/AECE.2015.0101

[18] G. Zames, "Feedback and Optimal Sensitivity: Model Reference Transformations Multiplicative Seminorms, and Approximate Inverses," IEEE Transaction on Automatic Control, vol. 26, no. 2, pp. 301-320, 1981. doi:10.1109/ TAC.1981.1102 603

[19] R. D. Martin, V. J. Yohai, R. H. Zamar, "Min-Max Bias Robust Regression," The Annals of Statistics, vol. 17, pp. 1608-1630, 1989. doi:http://dx.doi.org/10.1214/aos/1176347384

[20] D. Gu, "A Game Theory Approach To Target Tracking in Sensor Networks," IEEE Transaction on System Man and Cybernetics Part B, vol. 41, no. 1, pp. 2-13, 2011. doi:10.1109/TSMCB.2010.2040733

[21] J. C. Preisig, "Optimal Minmax Estimation and the Development of Minmax Estimation Error Bounds," IEEE International Conference on Acoustics, Speech, and Signal Processing, San Francisco, USA, 1992, pp. 285–288. doi:10.1109/ ICASSP.1992.226627

[22] I. Yaesh, U. Shaked, "Min–Max Kalman filtering," Systems and Control Letters, vol. 53, no. 3, pp. 217-228. 2004. doi:10.1016/j.sysconle.2004.04.008

[23] S. Zhuk, V. Mallet, "Reduced Minimax Filtering by Means of Differential-Algebraic Equations," 5th International Conference on Physics and Control, León, Spain, 2011, pp. 1-8.

[24] S. Zhuk, "Minimax State Estimation for Linear Stationary Differential-Algebraic Equations," in Proc. 16th IFAC Symposium on System Identification, Brussels, Belgium, 2012, pp. 143-148. doi:10.3182/20120711-3-BE-2027.00138

[25] I. Yaesh, U. Shaked, "Discrete-Time Min-Max Tracking," IEEE Transaction on Aerospace Electronic Systems, vol. 42, no. 2, pp. 540-547, 2006. doi:10.1109/ TAES.2006.1642570

[26] D. Simon, "A Game Theory Approach to Constrained Minimax State Estimation," IEEE Transaction on Signal Processing, vol. 54, no. 2, pp. 405-412. doi:10.1109/TSP.2005.861732

[27] M. Ghazal, A. Doustmohammadi, "A Novel Target Tracking Algorithm For Simultaneous Measurements of Radar and Infrared Sensors," Advances in Electrical and Computer Engineering, vol. 16, n. 3, 2016. doi:10.4316/AECE.2016.03009

[28] X.R. Li, V.P. Jilkov, "Survey of Maneuvering Target Tracking. Part I. Dynamic Models," IEEE Transactions on Aerospace and Electronic Systems, vol. 39, no. 4, pp. 1333-1364, 2003. doi:10.1109/TAES.2003.1261132

NOMENCLATURE

k	scan time index
j	system model index
$\bar{x}_k$	system state vector
y_k	measurement vector
$\mathbf{A}_j$	state transition matrix
$\mathbf{B}_j$	input matrix
$f(\bullet)$	measurement model
$\boldsymbol{R}^j$	process noise covariance matrix
$\boldsymbol{M}^j$	measurement noise covariance matrix
N	covariance matrix of n_k
S	set of all models
v_k^j	process noise vector
ω_k^j	measurement noise vector
$\delta_{k(k-i)}$	Kronecker delta function
$\tilde{M}_k^j$	*jth* system model at *kth* step
π_{ij}	transitions probability from *ith* model to *jth* model
φ	initial state distribution of Markov chain
$\boldsymbol{\mu}_k$	adversary steering command vector
μ_k^j	mode probability