

# Methods of Simulated Annealing and Particle Swarm Applied to the Optimization of Reactive Power Flow in Electric Power Systems

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**Abstract**—Electric power system is characterized by relatively high demand for lagging reactive power. From the economic viewpoint, reactive power sources should be installed close to its demand. Optimal compensation should ensure minimal costs of the reactive power generation and transmission within the considered system. The optimization of activities related to reactive power compensation concerns the location and power of compensation devices. This is to optimize voltage levels and reactive power flows in the system. The article presents methods of simulated annealing and particle swarm applied to solve an optimization task of the reactive power flow. It has been assumed that active power losses in a power system are the objective function.

**Index Terms**—optimization, heuristic algorithms, power systems, reactive power control, compensation.

## I. INTRODUCTION

From the economic viewpoint, reactive power sources should be installed close to its demand, which has a reducing effect on energy and power losses, voltage drops and the line transmission capability as well as an increasing effect on voltage levels, particularly there, where the voltage is low.

Transmission capability of a system is limited by apparent power, which means that the higher are reactive power flows the lower are flows of active power. The main sources of reactive power are synchronous generators and power lines – chiefly HV lines. Other reactive power sources that are purposefully installed in selected optimal locations are capacitor banks of the SVC type as well as static VAR compensators (SVC) and synchronous compensators (STATCOM). Presently, capacitors are the most often applied because of their simple design, usability, relatively low installation costs and a large selection of the equipment on the market. Compensators are less frequently used because they are more expensive.

Optimal compensation should ensure minimal costs of the reactive power generation and transmission within the considered system. Optimization of activities related to the compensation of reactive power concerns power and location of the compensation devices. The objective is to optimize levels of voltage and reactive power flows in the power system. Limitation of reactive power flows is another important question and it can be realized for instance by an adequate selection of nominal power for transformers, by shutting weakly loaded elements off and by avoiding idle operation of induction motors [1-2]. However,

the mentioned questions concern operating aspects of power devices and lines and as such are beyond the interest scope of the presented article.

## II. OPTIMIZATION OF VOLTAGE AND REACTIVE POWER AS AN OPTIMIZATION TASK

The history of power system optimization, both at the stage of the system operation control and of its development planning, is nearly as long as the history of solving the load flow tasks [3-4]. Full optimization (the costs and other) of a given operation state of a power system can be realized only when the transmission line operating conditions together with the related constraints are fully considered. The use of classical optimization methods is sometimes difficult or impossible due to the form of the objective function and its domain. The objective function is not always explicitly defined. Often, the range of the function domain is a set of integers, which is due to the specificity of the problem. Also, the derivative of the objective function is sometimes not known. Moreover, the objective function can be discontinuous at many points. In the course of calculations, some limitations can in some sense also be implicit, and when included in the objective function (the classical method of taking into account constraints), one cannot really tell what shape the new objective function thus created, and being optimized, takes.

In the case considered, the search space is partially discrete. A transformer's ratio of transformation changes in a stepwise manner, but in the case of capacitor banks, it is possible to switch on (or turn off) a particular module. We are dealing here with a certain discontinuity, which suggests the use of heuristic methods. This is also confirmed by other authors [5-8].

The considerations of the present article are based on the assumption that active power losses in a power system are the objective function. The literature and practice offer plenty alternatives to that, but such a solution has been accepted in order to arrive at the optimization objective formulation that would be clear and comprehensible from the engineering viewpoint right at the preliminary stage of the research. With such an approach, the OPF (*Optimal Power Flow*) task objective function can be written as follows [9]:

$$F_c(\mathbf{z}) = \Delta P(U_{G1} \dots U_{GN_z} v_{T1} \dots v_{TN_r} Q_{K1} \dots Q_{KN_k}) \quad (1)$$

$$F_c(\mathbf{z}) = \Delta P(U_G \mathbf{v}_T \mathbf{Q}_K) \quad (2)$$

where:

$U_G$  - vector of preset voltages in generator nodes,

$v_T$  - vector of the turns ratio of control transformers,

$Q_k$  - vector of reactive power of compensators installed in selected network system nodes,

$N_z$  - a number of generator nodes, where the voltage value is preset,

$N_r$  - a number of control transformers, where the turns ratio is changed,

$N_k$  - a number of compensators, where the value of reactive power is changed.

As can be seen, the objective function is relatively simple. It should be noted that with differently formulated objective functions the obtained control vectors can essentially differ from one another.

Equality and inequality constraints mostly follow from the problem scope of the load flow analysis and they include:

- power balance equations that should be valid for each network system node;
- equations that ensure balanced power exchange with abroad at the preset level;
- node-voltage constraints;
- constraints concerning control vector elements i.e. active and reactive power generated in the nodes;
- constraints that follow from the transmission capability of the network system elements (lines and transformers).

In this paper the following constraints have been assumed:

- maximal and minimal values of node voltages,
- maximal and minimal values of reactive power in the sources,
- allowed current rating for the branches,
- maximal and minimal values of reactive power in the compensators,
- maximal and minimal values of the turns ratio of control transformers,
- balance of the generated and load power in the considered area.

### III. METHODS FOR SOLVING THE ANALYZED OPTIMIZATION TASK

The discussed task can be solved with the use of a variety of methods from classical [4], [10] to heuristic ones [11-12].

The methods of computational intelligence allowed to increase the efficiency of solving particularly difficult and unusual computational tasks and problems for which the use of classical methods, fails.

Many heuristic optimization methods have been described in the literature, the inspiration for which was the observation of natural, physical and even sociological phenomena. An important part of these methods is detailed below:

artificial neural networks [13], fuzzy systems [14], evolutionary algorithms [15], simulated annealing [16], artificial immune systems [17], particle swarm optimization [18], ant algorithms [19], tabu search [20], bees algorithms [21], cuckoo search [22].

In addition to the listed algorithms, there are still other, more or less popular. Some of them, as mentioned above,

arise from the already existing, through appropriate assumptions about the control parameters.

In general, to summarize them all, it can be said that it is a class of methods designed to solve difficult, atypical, complex, multidimensional, discrete and not precisely defined problems. Their features, such as full randomness, calculations on n-vectors of variables simultaneously, or the possibility of modification during the course of calculations make them more and more popular. They are often resorted to in practice, when the use of classical methods is difficult or even impossible, or we want to quickly recognise the problem.

The present paper discusses the application of two heuristic methods that is simulated annealing and particle swarm. These two methods were chosen because they are characterized by very different algorithms and phenomena that inspired them.

The notion of simulated annealing refers to the annealing technique of metallurgy, which consists in heating and controlled cooling (gradual moving down to the states of lower energy) of steel. Realization of the optimization process with the use of the mentioned phenomenon consists in random selection of points from neighbourhood of the initial point and then finding a point, where the objective function assumes the lowest value. To get out of a local minimum, an increase of the objective function is accepted with a certain probability, i.e., if then the new state is accepted. Next, the "temperature decrease" occurs and the process gets started again. The probability of accepting a worse state is high at the beginning and decreases at the temperature decreases. The condition for the algorithm completion is to obtain a temperature value that is lower than the present value.

The algorithm procedure goes as follows [16]:

- Initialize – Start with a random initial placement, initialize a "temperature";
- Move – Perturbation the placement through a defined move;
- Calculate score – calculate the change in the score due to the move made;
- Choose – Depending on the change in score, accept or reject the move. The probability of acceptance depending on the current "temperature";
- Update and repeat – Update the temperature value by lowering the temperature. Go back to step 2.

The process is done until "Freezing Point" is reached.

Some of the most common criteria are based on

- the total number of solutions generated;
- the temperature at which the desired energy level is attained (freezing temperature);
- the acceptance ratio (ratio between the number of solutions accepted and the number of solutions generated).

The particle swarm method has been elaborated by James Kennedy and R. C. Eberhart in 1995 [18]. The algorithm has been developed based on the observation of social behaviour by certain kinds of animals such as birds and fish. Transferring the social behaviour concept onto the ground of mathematics, the assumption is that a single particle is a point or rather a vector composed of independent

variables, whose "positions" and "velocity" are adequately modified through successive iterations till the termination criterion is met and the optimal solution is obtained.

The algorithm procedure goes as follows [18]:

- individual particles are ascribed random initial positions and initial velocities;
- objective function is calculated for each particle;
- the best particle is selected and the best position for each of the particles gets remembered;
- velocity vector of each particle is updated considering the information of a given particle and a particle of the best known position, according to the below given dependence:

$$\mathbf{v}_{k+1}^i = w \mathbf{v}_k^i + c_1 r_1 (\mathbf{p}^i - \mathbf{x}_k^i) + c_2 r_2 (\mathbf{p}^g - \mathbf{x}_k^i) \quad (3)$$

where:

$\mathbf{x}_k^i$  - position vector of the i-th particle in the k-th iteration,  $\mathbf{v}_k^i$  - velocity vector of the i-th particle in the k-th iteration,  $\mathbf{p}^i$  - the best known position of the i-th particle,  $\mathbf{p}^g$  - the best known position found by the swarm leader,  $w$  - inertial coefficient of the particle movement,  $c_1, c_2$  - preset coefficients of acceleration or learning,  $r_1, r_2$  - numbers obtained with the random number generator of uniformly distributed within the interval of [0,1].

- the position of each particle is updated according to the dependence:

$$\mathbf{x}_{k+1}^i = \mathbf{x}_k^i + \mathbf{v}_{k+1}^i \quad (4)$$

In the formula (4), position and velocity are added, which can raise some doubts. However, it should be noted that the update of a particle position is performed after one iteration of the algorithm is done, so it is satisfied for  $t=1$  s and that is why it is this very form of it and no other that can be found in the literature.

- The stopping criterion is checked.

The stopping criterion often used in the literature is related to maximum number of iterations. The algorithm stops, if a maximum number of iterations is exceeded.

The particle swarm optimization algorithm belongs to a class of stochastic algorithms for global optimization and its main advantages are the easily parallelization and simplicity. In spite of the referred advantages, algorithm possesses some drawbacks, namely its parameters dependency and the slow convergence rate in the vicinity of the global minimum.

Both algorithm simulated annealing and particle swarm are very convenient to process optimization problem with continuous variables and discrete variables.

With heuristic methods it is not necessary to know the objective function derivative form. Their application goes undisturbed despite discontinuities of the function and when the calculation process gets "stuck" in a local minimum. For the discussed problem (load flow task) it is characteristic that a time-consuming iterative process has to be used to determine elements of the system state vector. Although the objective function in the form of a function of power losses is easy to optimize, one of the constraint groups concerning node constraints (allowed current rating and rated power of transformers) can be checked only on the basis of the system state vector that is difficult to determine. In a sense (in the calculation process), those constraints are

covert and when they get included to the objective function (classical method of considering the constraints) it is hard to decide what is the form of this new function, which is subject to minimization. Owing to the application of heuristic methods it is possible to avoid problems related to the form and nature of an optimization task that is the subject of the present paper.

The above problem has been solved with the application of a computer program to determine load flows in the network system. The PowerWorld software, ver. 13 has been used. The software includes the SimAuto add-on that enables connections with external applications (Matlab, Excel, Delphi), which have been mostly used for load-flow calculations. In order to solve the optimization task the Matlab environment has been used, where the script for the connection with the SimAuto add-on as well as for calling the algorithm of simulated annealing has been written [9]. This is shown in the Figure 1.

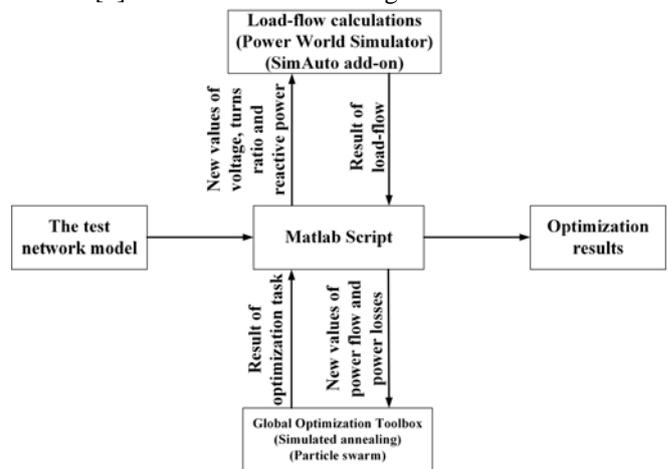


Figure 1. Wheel slip response with the proposed control algorithm

A modified test network CIGRE, also referred to as the C7M network system has been used for the testing purposes. This network system model has been developed in the mid 90-ties of the past century by research teams of the technology universities of Warsaw and Lublin (Figure 2). The original 5-node CIGRE network system has been the inspiration for the authors. The discussed model network consists of 30 nodes, 36 branches (lines and transformers), 33 loads, 8 sources (including one source that is connected to the balancing node) and three locations of capacitor banks.

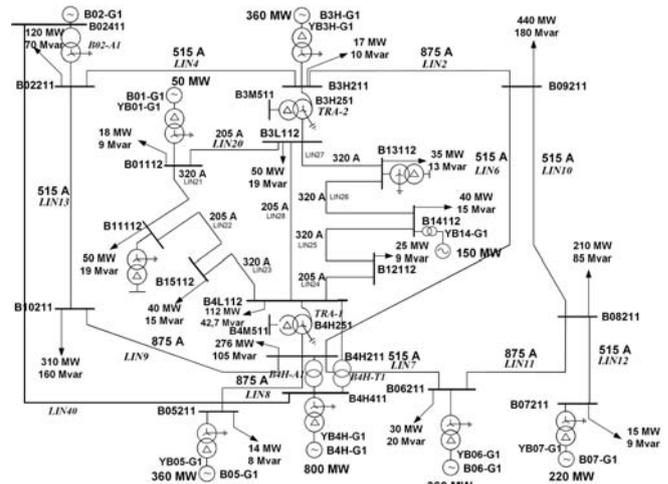


Figure 2. Diagram of the test network system CIGRE

Eleven below-given variables have been assumed for the optimization calculation purposes:

- pre-set voltage values in generating nodes (seven variables),

TABLE I. LIST OF NODES AND SOURCES WITH THEIR PARAMETERS

It.	Controlled node	Voltage rating	Voltage		Generator	Reactive power	
			Max.	Min.		Max.	Min.
-	-	kV	pu	pu	-	Mvar	Mvar
1	YB01-G1	10.5	1,118	0,955	B01_1-01	45	-10
2	YB05-G1	22	1,114	0,955	B05_2-01	190	-20,8
3	YB06-G1	22	1,114	0,955	B06_2-01	190	-20,8
4	YB07-G1	15.8	1,114	0,955	DO7_2-01	112,8	-21,9
5	YB14-G1	13.8	1,118	0,955	B14_1-01	86,8	-14,9
6	YB3H-G1	22	1,114	0,955	B3H_2-01	190	-20,8
7	YB4H-G1	27	1,05	0,95	B4H_4-01	661,8	-371,6

- turns ratio of coupling transformers (two variables),

TABLE II. LIST OF TRANSFORMERS TOGETHER WITH THEIR PARAMETERS

Item	Branch	Node Send.	Node Receiv.	Turns ratio	
				max	min
-	-	-	-	pu	pu
1	B4H-A1	B4H411	B4H211	1,1111	0,9091
2	B4H-T1	B4H411	B4L112	1,1364	0,8929

- reactive power in compensators (two variables).

TABLE III. LIST OF REACTIVE POWER SOURCES TOGETHER WITH THEIR PARAMETERS

Item	Node	Cap. bank	Reactive power	
			max	min
-	-	-	Mvar	Mvar
1	B3M511	B3M-C1	15	-15
2	B4M511	B4M-C1	24,3	-24,3

#### IV. CALCULATION RESULTS

Basic state of the CIGRE test network has been characterized by an exceeded voltage value in the node B09211. The voltage of that node, in the per-unit system, has been of 0.876 pu, while the allowed minimal voltage value for that node is 0,95. Active power losses at that state have been of 52,15 MW. Next, optimization calculations have been done for that test network system aiming at the minimization of power losses.

Results of the simulated annealing optimization and particle swarm optimization are set in the below given tables. Power losses have been reduced down to the value of 45,45 MW (particle swarm) and 45,7 MW (simulated annealing).

#### Results for simulated annealing

TABLE IV. LIST OF NODES AND SOURCES WITH THE PRESET VOLTAGE VALUE TOGETHER WITH THEIR PARAMETERS

Item	Controlled node	Optimized voltage	Generator	Reactive power optimized
-	-	pu	-	Mvar
1	B01112	1,091	B01_1-01	33,62
2	B05211	1,114	B05_2-01	95,39
3	B06211	1,114	B06_2-01	148,26
4	B07211	1,112	DO7_2-01	73,36
5	B14112	1,104	B14_1-01	59,46
6	B3H211	1,112	B3H_2-01	190
7	B4H411	1,04	B4H_4-01	428,33

TABLE V. LIST OF COUPLING TRANSFORMERS TOGETHER WITH THEIR PARAMETERS

Item	Branch	Node. Send.	Node. Receiv.	Turns ratio optimized
-	-	-	-	pu
1	B4H-A1	B4H411	B4H211	0,9096
2	B4H-T1	B4H411	B4L112	0,8931

TABLE VI. LIST OF REACTIVE POWER SOURCES TOGETHER WITH THEIR PARAMETERS

Item	Node	Cap. bank	Reactive power optimized
-	-	-	Mvar
1	B3M511	B3M-C1	14,43
2	B4M511	B4M-C1	11,02

#### Results for particle swarm

TABLE VII. LIST OF NODES AND SOURCES WITH THE PRESET VOLTAGE VALUE TOGETHER WITH THEIR PARAMETERS

Item	Controlled node	Optimized voltage	Generator	Reactive power optimized
-	-	pu	-	Mvar
1	B01112	1,073	B01_1-01	33,38
2	B05211	1,114	B05_2-01	95,05
3	B06211	1,114	B06_2-01	146,83
4	B07211	1,113	DO7_2-01	74,43
5	B14112	1,099	B14_1-01	59,21
6	B3H211	1,114	B3H_2-01	190
7	B4H411	1,03	B4H_4-01	427,82

TABLE VIII. LIST OF COUPLING TRANSFORMERS TOGETHER WITH THEIR PARAMETERS

Item	Branch	Node. Send.	Node. Receiv.	Turns ratio optimized
-	-	-	-	pu
1	B4H-A1	B4H411	B4H211	0,9091
2	B4H-T1	B4H411	B4L112	0,8929

TABLE IX. LIST OF REACTIVE POWER SOURCES TOGETHER WITH THEIR PARAMETERS

Item	Node	Cap. bank	Reactive power optimized
-	-	-	Mvar
1	B3M511	B3M-C1	15
2	B4M511	B4M-C1	10,65

## V. EVALUATION OF THE APPLIED OPTIMIZATION METHODS AND THE CALCULATION PROCESS

Results obtained using the two heuristic methods are slightly different. Realization of the simulated annealing method takes less time, but the obtained result is slightly worse as compared to the result obtained with the particle swarm method. This can be explained by the fact that the calculation process realized with the first method (simulated annealing) is based on one calculation point, while the other method (particle swarm) is based on  $n$  points, which makes the search for the optimum more efficient. The local best position (lbest) version of the particle swarm method has been used. The optimum search process by the lbest algorithm is slower, but it is more immune to getting stuck in a local optimum. In this case, finding the global solution is much more certain.

Efficiency of the simulated annealing method can be enhanced e.g. by the application of parallel calculations starting from different initial points. Each thus far found best solution gets stored during the calculation process, which guarantees that the actual best solution cannot be lost as a result of randomness in the process of the algorithm realization.

Figures 3 and 4 illustrate the computing process for the two discussed methods. The graphs show variability of the value of the function(s) in subsequent iterations.

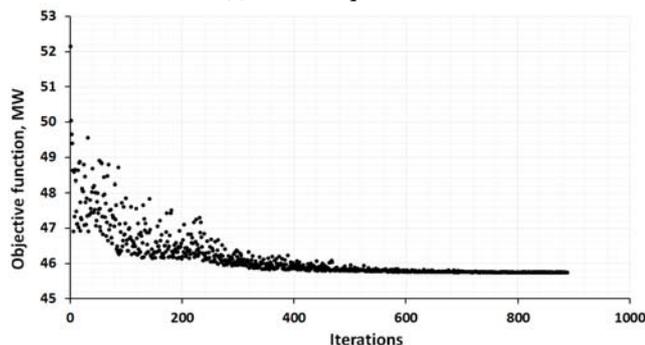


Figure 3. Objective function characteristic for the method of simulated annealing

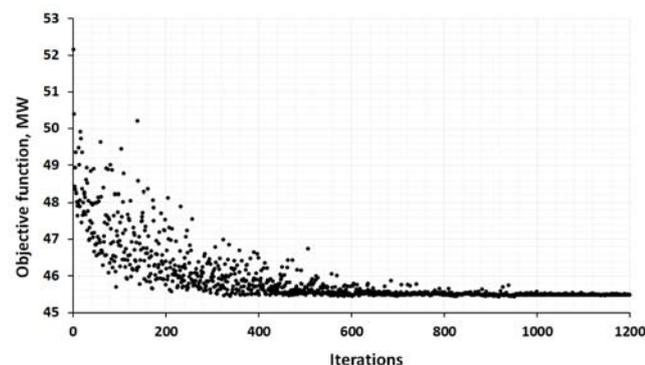


Figure 4. Objective function characteristic for the method of particle swarm

The above example concerns one calculated case. In order to verify efficiency of the both methods, calculations have been performed for thirty two variant cases. Repeatability of results obtained with data sets of different starting points has been tested by performing thirty two testing calculation runs, which have confirmed that the simulated annealing and the particle swarm algorithms find their solutions in the vicinity of the same point.

The solutions have been evaluated using the objective function variation ratio by the following dependence [23]:

$$V = \frac{s}{x_{av}} \quad (5)$$

where

$V$  – coefficient of variation;

$s$  – standard deviation obtained from the formula:

$$s = \sqrt{\sum_{i=1}^n (x_i - x_{av})^2} \quad (6)$$

$x_{av}$  – the mean calculated by the formula:

$$x_{av} = \frac{1}{n} \sum_{i=1}^n x_i \quad (7)$$

For the simulated annealing method, the standard deviation is 0.09, while in the case of the particle swarm method its value amounts to 0.05.

In the case of the simulated annealing method, variation ratio of the objective function (power losses) is 0.002, while the respective value for the particle swarm method is 0.001. The below diagram shows variation of the objective function for the both discussed methods by 32 calculated variant cases.

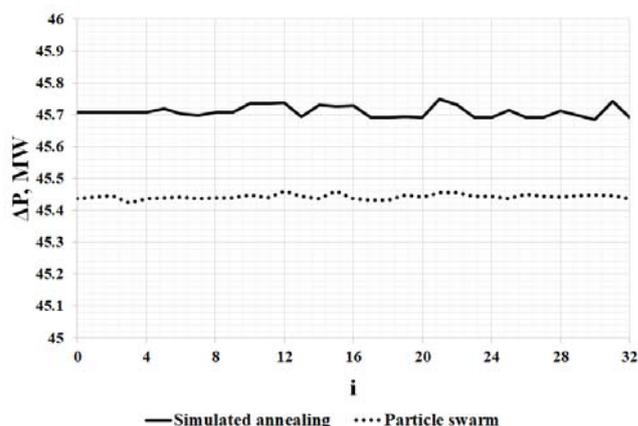


Figure 5. Variation of the objective function in 32 calculated variant cases

## VI. OPTIMAL LOCATION OF ADDITIONAL REACTIVE POWER SOURCES

Additional sources of reactive power in the form of capacitor or compensator banks can be installed in various places of the network, depending on the needs.

Four criteria were considered for the calculation using the CIGRE test network:

**V-Q stability criterion**, which consists in the fact that in the successive steps a small voltage reduction in a selected node is assumed, and for such a state, reactive power is determined in this node, necessary to maintain the assumed voltage. If the reduction in the node voltage corresponds to the reduction of the reactive power of the node, the network is considered to be stable. If the reduction of the node voltage in the successive steps is no longer accompanied by a decrease, but an increase in reactive power of the node, it is assumed that the network is unstable. The voltage at which the reactive power increase changed the sign to the opposite is called the "critical voltage", and the reactive power of the node at which this occurred is called the "limit power".

**P-V stability criterion**, which consists in the fact that the active power consumption is tested as a function of the voltage in the node. P-V curves are determined for selected network nodes using the "step by step" method. In the next steps, a small load of the node is assumed with active power and determines the voltage in this node. The process ends when the calculations of power flows are divergent, which means that at a given point there is a loss of stability in the network.

**Criterion for testing voltage changes due to changes in reactive power**, which consists in increasing the reactive power consumption in selected nodes of the network and determining the voltage drop associated with it. The nodes in which the voltage drop is the highest are eligible for the installation of reactive power sources there.

**The criterion of changes in active power losses due to changes in reactive power**, which consists in increasing the reactive power consumption in selected nodes of the network and determining the percentage difference in active power losses in the whole network in relation to active power losses in the ground state. Nodes in which the increase in reactive power consumption will result in the highest percentage changes in active power losses in the network are eligible for the installation of reactive power sources there.

Taking into account the results of calculations based on all criteria, it was assumed that new sources of reactive power will be connected to nodes: B08211, B09211, B10211, B11112, B13112, B3L112, B4L112, B4H211.

The results of the optimization are presented in the table below. Power losses in the network for the simulated annealing method amounted to 36.13 MW and for the particle swarm method to 36.08 MW.

TABLE X. LIST OF REACTIVE POWER SOURCES

Item	Node	Cap. bank	Reactive power optimized	
			simulated annealing	particle swarm
-	-	-	Mvar	Mvar
1	B08211	B08-C1	70,8	77,9
2	B09211	B09-C1	168,6	160,1
3	B10211	B10-C1	135,2	143,7
4	B11112	B11-C1	18,6	20,0
5	B13112	B13-C1	25,5	32,9
6	B3L112	B3L-C1	-76,6	-52,8
7	B4L112	B4L-C1	-5,3	-26,4
8	B4H211	B4H-C1	144,0	142,0

## VII. CONCLUSION

Development of the power industry, an ever increasing number of loads of various time characteristics as well as of the sources makes it necessary to apply new flexible adaptive methods for the real-time control of the power system operation. Reactive power management is important for a power network system. Undesired flows of reactive power have a negative effect on the values of node voltages, voltage drops, active power losses and cause reduced transmission capacity of the power system elements. Intuitive control of reactive power can be successful but only in the case of small network systems. When extensive power systems are concerned, it is necessary to apply algorithms of solid mathematical base that are characterized

by high efficiency and fast performance. The article uses two heuristic methods which allowed for effective solution of the presented problem. They can be easily implemented using available software, Matlab and PowerWorld Simulator. The combination of these two tools allow the use of various optimization methods in power engineering.

## REFERENCES

- [1] A. Meier, "Electric Power Systems: Conceptual Introduction", pp. 144–228, Wiley-IEEE Press, 2006.
- [2] J. Machowski, J. Bialek, J. Bumby, "Power system dynamics stability and control", pp. 15–122, John Wiley & Sons, 2008.
- [3] L. L. Grigsby, "Power systems", pp. 46–56, CRC Press, 2012.
- [4] J. Zhu, "Optimization of Power System Operation", pp. 1–50, Wiley-IEEE Press, 2015.
- [5] W. Zhang, Y. Liu, "Multi-objective reactive power and voltage control based on fuzzy optimization strategy and fuzzy adaptive particle swarm," International Journal of Electrical Power & Energy Systems, vol. 30, no. 9, pp. 525–532, 2008. doi:10.1016/j.ijepes.2008.04.005.
- [6] C. Wang, G. Yao, X. Wang et al., "Reactive Power Optimization Based on Particle Swarm Optimization Algorithm in 10kV Distribution Network," Advances in Swarm Intelligence, vol. 6728, pp. 157–164, 2011. doi: 10.1007/978-3-642-21515-5\_19.
- [7] A. Q.H. Badar, B. S. Umre., A. S. Junghare, "Reactive power control using dynamic Particle Swarm Optimization for real power loss minimization," International Journal of Electrical Power & Energy Systems, vol. 41, no. 1, pp. 133–136, 2012. doi:10.1016/j.ijepes.2012.03.030.
- [8] S. Biswas, K. K. Manadal, N. Chakraborty, "Simulated Annealing Based Real Power Loss Minimization Aspect for a Large Power Network," Swarm, Evolutionary, and Memetic Computing, vol. 8297, pp. 345–353, 2013. doi: 10.1007/978-3-319-03753-0\_31.
- [9] M. A. Abido, "Multiobjective Optimal VAR Dispatch Using Strength Pareto Evolutionary Algorithm," 2006 IEEE International Conference on Evolutionary Computation, Vancouver, pp. 16–21, 2006. doi: 10.1109/CEC.2006.1688384.
- [10] M. S. Bazaraa, H. D. Sherali, C. M. Shetty, "Nonlinear Programming: Theory and Algorithms", pp. 1-313, Wiley, 2006.
- [11] Z. Michalewicz, D. B. Fogel, "How to Solve It. Modern Heuristics", pp. 145-487, Springer, 2004.
- [12] X.-S. Yang, "Nature-inspired metaheuristic algorithms", pp. 11-108, Luniver Press, 2010.
- [13] P. J. Braspenning, F. Thuijsman, A. J. M. M. Weijters, "Artificial Neural Networks: An Introduction to ANN Theory and Practice", pp.1–100, Springer, 1995.
- [14] W. Pedrycz, "Fuzzy control and fuzzy systems", pp. 1–78, John Wiley & Sons, 1996.
- [15] L. J. Fogel, Owens, A., J., M., J. Walsh, "Artificial Intelligence through Simulated Evolution", pp. 11-66, John Wiley & Sons, 1966.
- [16] S. Kirkpatrick, C. D. Gelatt, M. P. Vecchi, "Optimization by Simulated Annealing," Science, vol. 220, no. 4598, pp. 671–680, 1983. doi: 10.1126/science.220.4598.671.
- [17] H. Bersini, J. Varela, Francisco, "Hints for adaptive problem solving gleaned from immune networks," Parallel Problem Solving from Nature, vol. 496, pp. 343–354, 1990. doi: 10.1007/BFb0029775.
- [18] R. Eberhart, J. Kennedy, "A new optimizer using particle swarm theory," in MHS'95. Proceedings of the Sixth International Symposium on Micro Machine and Human Science, pp. 39–43, IEEE, 1995. doi: 10.1109/MHS.1995.494215.
- [19] A. Colomi, M. Dorigo, V. Maniezzo, "Distributed Optimization by Ant Colonies," Appeared in Proceedings of ECAL91, pp. 134–142, 1991.
- [20] F. Glover, "Tabu Search—Part I," ORSA Journal on Computing, vol.1, no. 3, pp. 190–206, 1989. doi: 10.1287/ijoc.1.3.190.
- [21] D. T. Pham, A. Ghanbarzadeh, E. Koc et al., "The Bees Algorithm A Novel Tool for Complex Optimisation," Intelligent Production Machines and Systems, pp. 454-459, 2006, doi: 10.1016/B978-008045157-2/50081-X.
- [22] X.-S. Yang, S. Deb, "Cuckoo Search via Lévy flights," 2009 World Congress on Nature & Biologically Inspired Computing, pp. 210–214, 2009. doi:10.1109/NABIC.2009.5393690.
- [23] J. E. Freund, B. M. Perles, "Modern Elementary Statistics", pp. 43-93, Pearson, 2006.