An Improved Analytical Methodology for Joint Distribution in Probabilistic Load Flow

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***Abstract*—This paper presents a novel analytical method based on improved Gaussian mixture model (GMM) to solve the probabilistic load flow problem. The proposed method accounts for the uncertainty introduced due to increasing percentages of renewable generation. First, the joint probability density function of several wind farms outputs is derived by using the improved GMM with the estimated parameters obtained by genetic algorithm (GA) in this paper, which could improve the accuracy of the probabilistic model. Next, the analytical expressions between the output power of wind farms and line power of power system are deduced by linearizing load flow equations. And, the joint probability density function and joint cumulative distribution function of line power are obtained from linear load equation and joint probability density function of wind output power. Finally, the proposed method, Monte Carlo simulation (MCS) and traditional GMM based methods are all tested on a modified IEEE 39-bus system and a modified IEEE 118-bus system with multiple wind farms, which demonstrates the feasibility of the proposed method.**

***Index Terms*—gaussian mixture model, maximum likelihood estimation, genetic algorithm, density function, distribution.**

I. Introduction

The random nature of power systems is accentuated with the increased penetration of large-scale renewable generation, such as wind farms and photovoltaic systems. Probabilistic load flow (PLF) is a more reliable approach for analyzing power systems considering the random nature and behavior of renewable generation, in comparison to the deterministic load flow methods.

PLF was first proposed by Borkowska in 1974 [1]. With the PLF theory developing, there are a few methods for PLF [2-13]. At present, they are mainly including simulation method, the point estimation method and the cumulant method. Simulation method represented by Monte Carlo simulation (MCS) is generally accurate and is often used as a standard for testing other methods for PLF. The basic idea of point estimation method [2] is to generate deterministic sample points and their weights based on the first moments of each input random variable, and use these sample points and weights to find the moments of the output random variables. The advantage of this method is that the required samples are small and the calculation speed is fast. However, because only the information of the few front moments of the input random variable is used (such as the three-point estimation method utilizes the four order moments), there is a certain degree of error in the high order calculation of the output random variables.

PLF based on cumulant method uses algebraic operations instead of convolution operations, which has the advantages of simple calculations and fast calculations speed. Its specific calculation process [3] is: calculate the cumulant of each order according to the central moment of the input random variable; based on Taylor series expansion, the power flow equation is linearized at the reference operating point, and then the cumulants of the state variables such as the node voltage and the branch current are calculated. Finally, the probability density distribution of the state variables is fitted by Gram-Charlier series expansion, and the cumulative distribution curve can be obtained. However, after the introduction of large-scale wind power, the skewness and kurtosis of the state variables are likely to exceed a certain range, which limits the accuracy of the conventional Gram-Charlier series. Therefore, improved Gram-Charlier series was developed in [14], and the Beta distribution was used to describe the wind power fluctuation characteristics. However, wind power fluctuation cannot be fairly modeled by single probabilistic model, especially for non-Gaussian correlated random power variables. Besides, most current literatures focus on marginal distribution of active power on multiple transmission lines. Actually, it is the joint distribution of multiple transmission lines, rather than marginal distribution, that can provide more comprehensive and precise security assessment of power system [15].

Based on the above analysis, this paper proposes a novel analytical method based on improved Gaussian mixture model (GMM) to solve PLF considering random nature of renewable generation, which can solve joint distribution of multiple transmission lines to assess the security of power systems. GMM is an effective way to represent fluctuation characteristics of output power of renewable generation. Compared with single distribution models such as Weibull distribution and Beta distribution [13-14], GMM gives higher precision [15]. Obtaining GMM's parameters is regarded as a maximum likelihood estimation problem, which can be solved by well-known expectation maximization (EM) algorithm [18-19]. However, EM algorithm is greatly impacted by the initial value, which

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might lead it to give sub-optimal results. To overcome the disadvantages of EM algorithm, this paper proposes a method based on genetic algorithm (GA) to solve for GMM's parameters. The linear analytical expressions between output power of several wind farms and power on several transmission lines can be deduced by linearizing the load flow equation. Therefore, the joint probability density function and the joint cumulative distribution function of line power can be correspondingly obtained.

The rest of the paper is structured as follows. In Section II, the basic theory of GMM is introduced. The fluctuation characteristics of actual wind power are modeled by GMM, and parameters of GMM are solved by GA. In Section III, the linear analytical expressions of PLF are derived. The joint probability density function and joint cumulative distribution function of line power is obtained. Modified IEEE 39-bus and 118-bus system are used as the test system for this analysis as shown in Section IV. Section V concludes this paper.

II. Gaussian Mixture Model

A. Basic theory of Gaussian Mixture Model

Gaussian Mixture Model (GMM) is a model that represents the mixed density distribution, and has been extensively used in pattern recognition and machine learning. It combines finite Gaussian distributions to model different types of non-Gaussian random variables. Thus, the GMM is used to model wind power output of several wind farms to characterize the wind power fluctuation characteristics in this paper. Its probability density functions are shown as follows,

$$f_{\tilde{\mathbf{X}}}(\mathbf{X}) = \sum_{m=1}^{M} \omega_m N_m(\mathbf{X}|\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m), \sum_{m=1}^{M} \omega_m = 1 \quad (1)$$

$$N_m(\mathbf{X}|\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m) = \frac{exp\left(-\frac{1}{2}(\mathbf{X}-\boldsymbol{\mu}_m)^T \boldsymbol{\Sigma}_i^{-1}(\mathbf{X}-\boldsymbol{\mu}_m)\right)}{(2\pi)^{W/2}\det(\boldsymbol{\Sigma}_m)^{W/2}} \quad (2)$$

where, $\mathbf{X}$ is input variable vector, representing output power of several wind farms. ω_m is weight coefficient of the m^{th} Gaussian component, which belongs to [0,1]; $\boldsymbol{\mu}_m$ and $\boldsymbol{\Sigma}_m$ are mean and variance (or mean vector and covariance matrix in multivariate case) of the m^{th} Gaussian component. Parameters to be estimated are ω_m $\boldsymbol{\mu}_m$ and $\boldsymbol{\Sigma}_m$, which can be solved by parameter maximum likelihood estimation (MLE) method. The MLE problem can be solved by well-known Expectation Maximization (EM) algorithm [16-17]. The basic idea is to iteratively estimate the model parameters by re-estimating the formula, leading the likelihood function to maximum. Given $X = \{X_1, \cdots, X_N\}$ representing sample of output power of wind farms, the likelihood function can be developed as shown in (3) and (4).

$$\mathrm{L}(X_1, \cdots X_N) = \prod_{j=1}^{N} f(X_j) \quad (3)$$

$$\ln L(X_1, \cdots X_N) = \sum_{j=1}^{N} \ln \sum_{m=1}^{M} \omega_m N_m(X_j) \quad (4)$$

B. Genetic Algorithm (GA)

The estimated parameters of GMM describing the wind farms outputs can be obtained by EM algorithm. However, EM algorithm is affected by the initial value, which might make it converge to the local optimal solution instead of the global optimal solution. GA has less dependence on the initial value of the problem with better global search ability [20]. It can quickly search out the entire solution in the solution space without falling into local optimal solution. Therefore, GA will be used in this paper instead of EM algorithm to solve MLE problem.

The core idea of GA is the survival of the fittest. By evaluating the fitness of each individual (likelihood function of wind power), it judges whether the individual should be inherited or eliminated, and then a global optimal solution can be determined. The specific calculation process of GA with the help of likelihood function is developed according to [20].

III. Analytic Method Based on Improved GMM

A Analytical Expressions

Linearization method of the power flow equation has been extensively used for probabilistic load flow [13]. The linear expressions are obtained by spreading the power flow equation at the reference point in Taylor series and ignoring higher order items greater than one:

$$\begin{aligned} \mathbf{U} &= \mathbf{U}_0 + \Delta\mathbf{U} = \mathbf{U}_0 + \mathbf{S}_0\Delta\mathbf{X} \\ \mathbf{W} &= \mathbf{W}_0 + \Delta\mathbf{W} = \mathbf{W}_0 + \mathbf{T}_0\Delta\mathbf{X} \end{aligned} \quad (5)$$

where $\mathbf{U}$ is the voltage amplitude and phase angle vector, $\mathbf{X}$ is the output power of wind farms, $\mathbf{W}$ is the power on transmission lines, the subscript 0 represents the reference point, $\mathbf{S}_0$ is the inverse matrix of Jacobin matrix, where $\mathbf{T}_0=\mathbf{G}_0\mathbf{S}_0$, $\mathbf{G}_0 = \partial\mathbf{W} / \partial\mathbf{U}\big|_{\mathbf{X}=\mathbf{X}_0}$, $\Delta\mathbf{X} = \mathbf{X} - \mathbf{X}_0$

Vector $\mathbf{U}_0$, $\mathbf{W}_0$ and Jacobin matrix can be obtained through deterministic load flow calculation, and then $\mathbf{T_0}$ and $\mathbf{S}_0$ can be obtained correspondingly. To better introduce following contents, a general equation formula that represents the linear relationship between wind power output $\mathbf{X}$ and power on transmission lines $\mathbf{W}$ derived from equation (5) is given in (6), where $\mathbf{T}_0=\mathbf{B}$, $\mathbf{C} = \mathbf{W}_0 - \mathbf{T}_0\mathbf{X}_0$.

$$\mathbf{W} = \mathbf{B}\mathbf{X} + \mathbf{C} \quad (6)$$

B. Probability distribution of power on transmission lines

In probability theory, the linear transformation of random variables obeying Gaussian distribution still obeys Gaussian distribution. When wind farms output $\mathbf{X}$ is modeled by the multivariate Gaussian distribution $N_m(\mathbf{X})$, the transmission lines power $\mathbf{W}$ that is the linear transformation of $\mathbf{X}$ as indicated in (6) obeys multivariate Gaussian distribution with mean vector $\mathbf{B}\boldsymbol{\mu}_m + \mathbf{C}$ and covariance matrix $\mathbf{B}\boldsymbol{\Sigma}_m\mathbf{B}^{\mathrm{T}}$. Therefore, the joint PDF of $\mathbf{W}$ is given as follows:

$$\begin{aligned} N_m(\mathbf{W}|\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m) &= \frac{1}{(2\pi)^{K/2}\det\left(\mathbf{B}\boldsymbol{\Sigma}_m\mathbf{B}^T\right)^{1/2}} \\ &\times e^{\left(-\frac{1}{2}(\mathbf{W}-\mathbf{B}\boldsymbol{\mu}_m-\mathbf{C})^T \boldsymbol{\Sigma}_i^{-1}(\mathbf{W}-\mathbf{B}\boldsymbol{\mu}_m-\mathbf{C})\right)} \end{aligned} \quad (7)$$

where K represents the number of transmission lines. Therefore, there is no need to use the series expansion method [4] to fit the probability distribution of lines power. It is only necessary to calculate the mean vector and variance matrix of wind power outputs by the improved

GMM, and then the mean vector and variance matrix of lines power can be determined by (7), and then the corresponding probability distribution can be developed.

According to (7), the joint cumulative distribution function (CDF) of **W** can be obtained by several integrals:

$$F_m(\mathbf{W}) = \int \cdots \int_{-\infty}^{W} N_m(\mathbf{W}|\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m) \mathrm{d}W_1 \cdots \mathrm{d}W_k \tag{8}$$

According to (1), when the probability density of output power **X** is modeled by GMM, according to total probability formula (9), the joint CDF of transmission lines power **W** can be obtained as shown in (10):

$$p(\mathbf{W}) = \sum_{m=1}^{M} p(\mathbf{W}/M) \times \omega_m \tag{9}$$

$$\begin{aligned} F(\mathbf{W}) &= \int \cdots \int_{-\infty}^{W} \sum_{m=1}^{M} \omega_m N_m(\mathbf{W}|\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m) dW_1 \cdots dW_K \\ &= \sum_{m=1}^{M} \omega_m \left[\int \cdots \int_{-\infty}^{W} N_m(\mathbf{W}|\boldsymbol{\mu}_m, \boldsymbol{\Sigma}_m) dW_1 \cdots dW_K \right] \\ &= \sum_{m=1}^{M} \omega_m F_m(\mathbf{W}) \end{aligned} \tag{10}$$

Correspondingly, the joint PDF can be obtained by differentiating (10):

$$\begin{aligned} f(\mathbf{W}) &= \frac{\partial^K}{\partial \mathbf{W}_1 \cdots \partial \mathbf{W}_K} \sum_{m=1}^{M} \omega_m F_m(\mathbf{W}) \\ &= \sum_{m=1}^{M} \omega_m \frac{\partial^K}{\partial \mathbf{W}_1 \cdots \partial \mathbf{W}_K} F_m(\mathbf{W}) = \sum_{m=1}^{M} \omega_m N_m(\mathbf{W}) \end{aligned} \tag{11}$$

Compared with marginal distribution, the joint CDF and joint PDF as indicated in (10) and (11) can provide more comprehensive and exact assessment for security of power system. In conclusion, the proposed method in this paper can be summarized as shown Fig. 1.

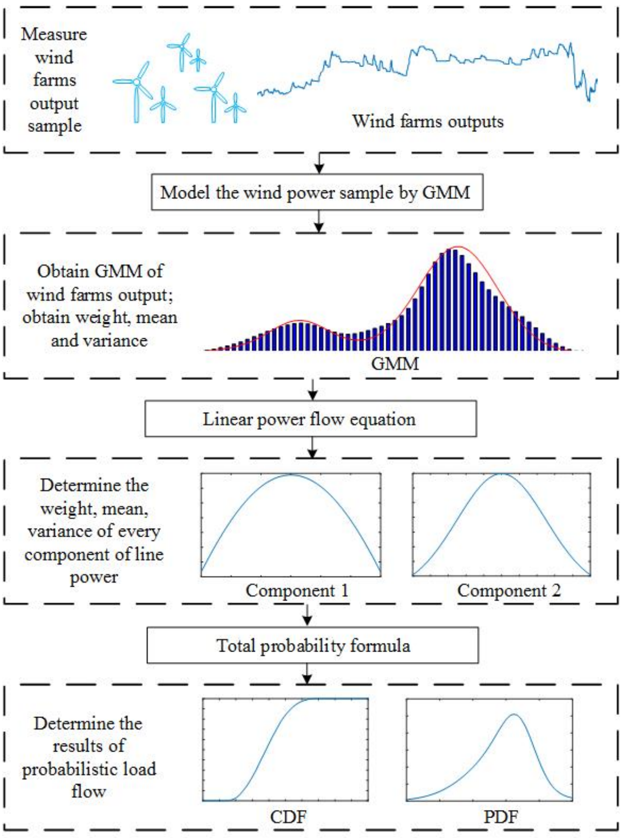

Figure 1. Calculation process of the proposed method

IV. PERFORMANCE ANALYSIS

A. Comparisons of algorithms

In this section, Gaussian Mixture Model (GMM) is modeled by the EM and GA respectively based on three month's actual wind power output in a certain region. This wind power sample was measured by PMUs, and was sampled at 10 minute intervals. The probability density function (PDF) is shown in Fig. 2, which is based on a normal kernel function [21].

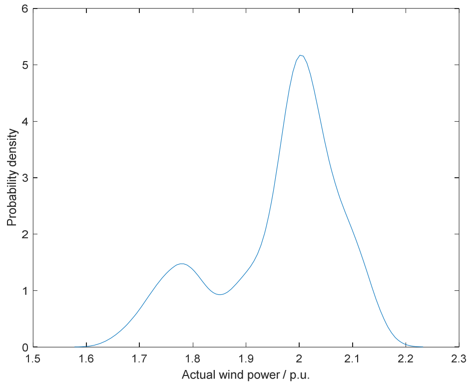

Figure 2. PDF of the actual wind power

Then, the GMM with 3 components of wind power is obtained by the EM algorithm and GA respectively, as is shown in Fig. 3.

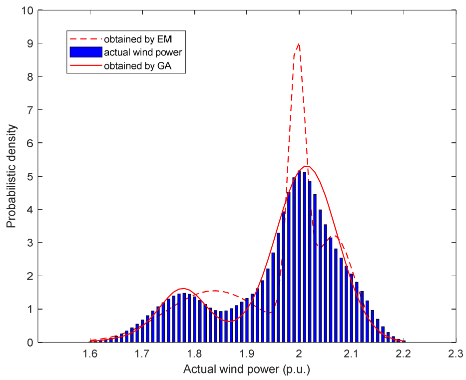

Figure 3. The comparison of results

In Fig. 3, the histogram represents the probability density of the data sample obtained by normal kernel function; the dotted line is the probability density obtained by the EM algorithm; the solid line is the probability density obtained by the GA improvement. Intuitively, results of EM algorithm have a large deviation from the actual data, especially on the abscissa 1.8-2.1, and the probability density obtained by GA matches very closely with the actual wind power. In order to quantitatively compare fitting effect between the two methods, the average root mean square (ARMS) and maximal absolute error (MAE) are used to verify the higher accuracy of the proposed method. *ARMS* and *MAE* are defined as:

$$\begin{aligned} ARMS &= \frac{\sqrt{\sum_{i=1}^{N} (MS_i - GM_i)^2}}{N} \\ MAE &= max\{|MS_i - GM_i|\}_{i=1,\cdots,N} \end{aligned} \tag{12}$$

where MS_i and GM_i denote the i^{th} point on the PDF curve obtained by EM and GA, and N represents the number of points. The ARMS and MAE obtained by EM are 0.0161 and 4.2368, and ARMS and MAE obtained by GA are 0.0011 and 0.7331, which verifies that the fitting effect of GA is better than EM algorithm and the ability of GA to seek optimal solutions is indeed better in maximum likelihood estimation. Thus, GMM can precisely represent probability density of unknown distribution.

B. Modified IEEE 39-bus System

The proposed probabilistic load flow method is tested on a modified IEEE 39-bus system to demonstrate its feasibility and effectiveness. The modified IEEE 39-bus system is shown in Fig. 4. Since the location of wind farms do not affect the results obtained by the proposed method, two wind farms named WF1 and WF2 are assumed to be integrated at bus 30 and bus 35 arbitrarily. There are no available wind power output samples for two wind farms. Thus, without losing generality, the data sample of two wind farms outputs $\mathbf{X}$ are constructed by the Nataf technique [10], under the assumption that the marginal distributions of $\mathbf{X}$ obey Beta distributions. Other detailed parameters of the system can be found in [22]. In the following test, the Monte Carlo simulations (MCS) are used as test standard to demonstrate the proposed method. Meanwhile, the method based on GMM modeled by EM is also simulated to compare with the proposed method improved by GA.

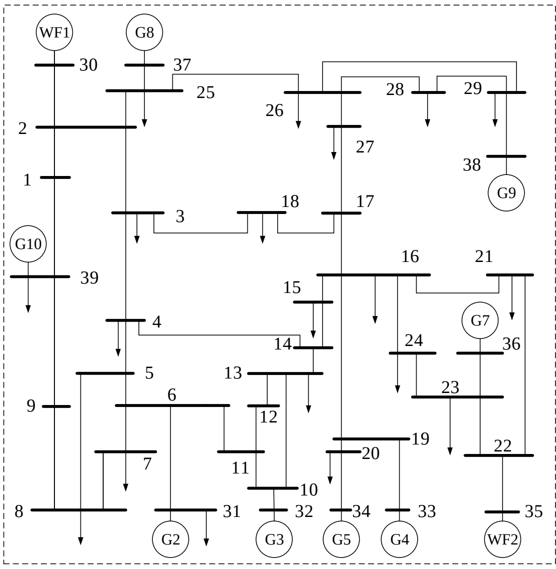

Figure 4. Modified IEEE 39-bus system

Two wind farms output $\mathbf{X}$ is modeled by 2 Gaussian components GMM with the help of GA to determine its joint probability density function (JPDF). The parameters of GMM are shown in table I, and the figure of JPDF of $\mathbf{X}$ is shown in Fig. 5. Meanwhile, the wind power output is also modeled by EM based GMM shown in table II.

TABLE I. PARAMETERS OF GMM OBTAINED BY GA

$$\omega_1=0.5571,\ \boldsymbol{\mu}_1=\begin{bmatrix}0.3378\\0.6186\end{bmatrix},\ \omega_2=0.4429,\ \boldsymbol{\mu}_2=\begin{bmatrix}0.5430\\0.8344\end{bmatrix}$$

$$\boldsymbol{\Sigma}_1=\begin{bmatrix}0.0186 & 0.0138\\0.0138 & 0.0209\end{bmatrix},\ \boldsymbol{\Sigma}_2=\begin{bmatrix}0.0219 & 0.0086\\0.0086 & 0.0065\end{bmatrix}$$

TABLE II. PARAMETERS OF GMM OBTAINED BY EM

$$\omega_1=0.4627,\ \boldsymbol{\mu}_1=\begin{bmatrix}0.6445\\0.9368\end{bmatrix},\ \omega_2=0.5373,\ \boldsymbol{\mu}_2=\begin{bmatrix}0.4003\\0.7204\end{bmatrix}$$

$$\boldsymbol{\Sigma}_1=\begin{bmatrix}0.0323 & 0.0128\\0.0128 & 0.0196\end{bmatrix},\ \boldsymbol{\Sigma}_2=\begin{bmatrix}0.0328 & 0.0256\\0.0256 & 0.0307\end{bmatrix}$$

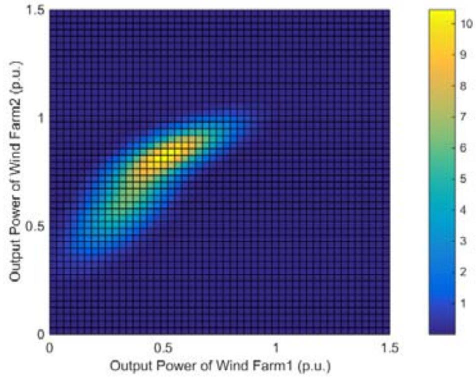

Figure 5. JPDF of two wind farms output

The number of lines does not affect the results obtained by the proposed method. Hence, firstly, oneline power between buses 2 and 3 is randomly selected to determine its PDF and CDF. Linear relationship between the wind farms output and line power can be obtained shown in (13) by linearizing method mentioned in (6).

$$\mathbf{B}=\begin{bmatrix}0.6283 & -0.0118\end{bmatrix},\ C=2.1261 \qquad (13)$$

Thus, according to the linear transformation invariance of Gaussian distribution and JPDF of wind farms output obtained by EM and GA, the PDF and CDF of selected line power can be determined as shown in Fig. 6 and 7.

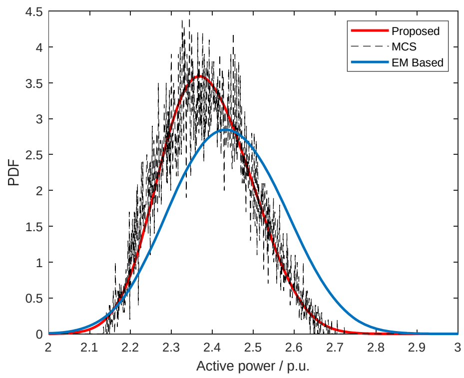

Figure 6. PDF of one-line power in modified 39 bus system

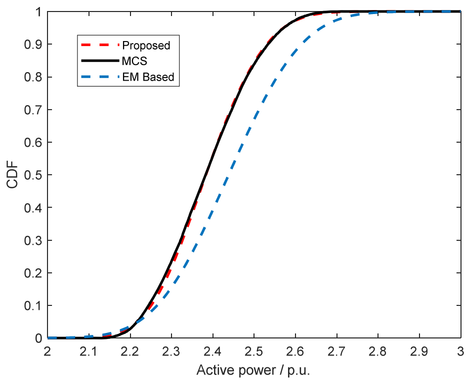

Figure 7. CDF of one-line power in modified 39 bus system

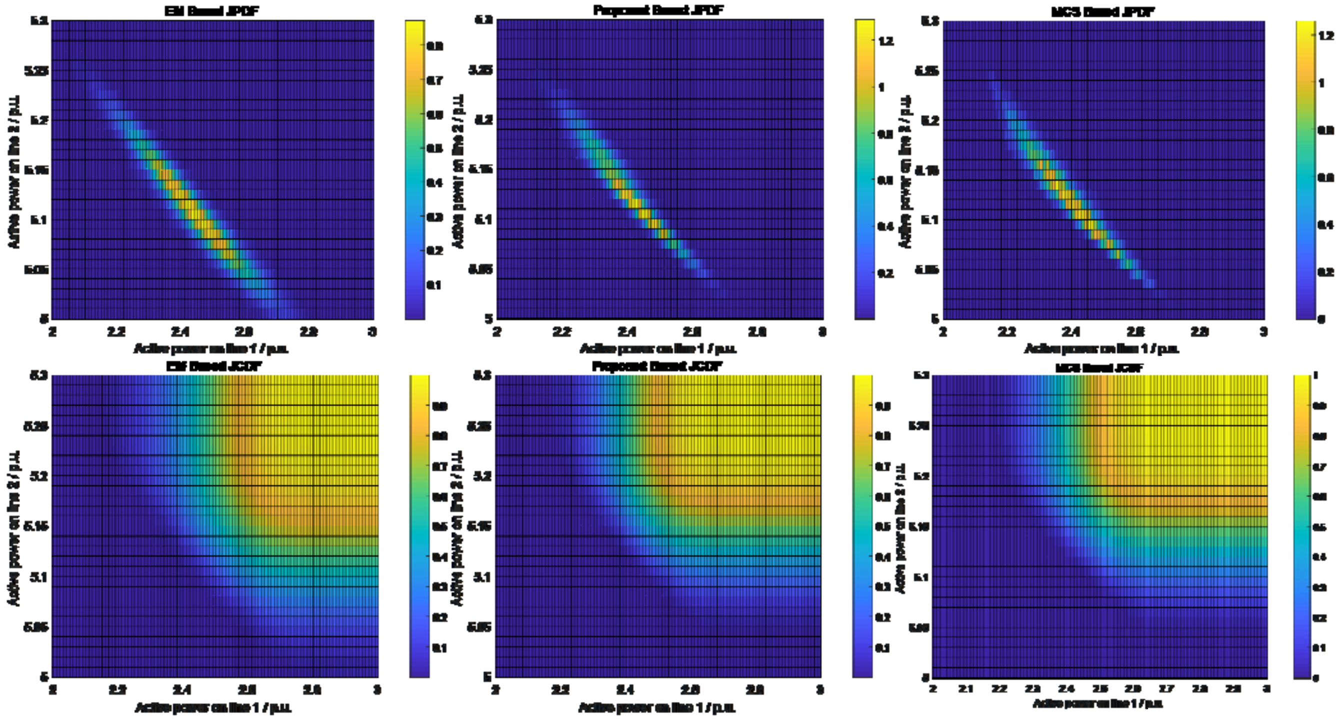

Figure 8. JCDF and JPDF of two lines in modified 39-bus system with two wind farms

The weight, mean and variance obtained by EM and GA respectively are shown in table III. In Fig. 6 and 7, the blue line and red line are respectively obtained by EM and GA; the black line is obtained by MCS with 10000 iterations. The blue lines obtained by EM have large deviation from black lines obtained by MCS. However, the red lines obtained by the proposed method match well with black lines. In Fig. 7, the ARMSs of the proposed method and the method based on EM are 0.00019 and 0.027 respectively, and the MAE of the proposed method and the method based on EM are 0.019 and 0.18. Hence, it can be concluded that the proposed method gives higher accuracy for PLF results.

TABLE III. PARAMETERS OF LINE POWER

$\omega_1=0.5571, \mu_1=2.3302, \Sigma_1=0.0071$
$\omega_2=0.4429, \mu_1=2.4551, \Sigma_2=0.0087$

Then, to further verify the feasibility of the proposed method, the active power on two transmission lines between buses 2, 3 and 6, 7 are randomly selected to determine their JPDF and joint cumulative density function (JCDF). Transmission line between buses 2 and 3 is recorded as line 1, and transmission line between buses 6 and 7 is recorded as line 2. By linearizing the power flow equation according to (5), the linear relationship (14) between line power on two transmission lines and two wind farms outputs can be obtained:

$$\mathbf{B}=\begin{bmatrix}0.6283 & -0.0118\\ -0.1575 & -0.1074\end{bmatrix}\quad \mathbf{C}=\begin{bmatrix}2.1261\\ 5.2707\end{bmatrix}\tag{14}$$

Thus, the JPDF and JCDF of two lines power obtained by EM and GA can be obtained as shown in Fig. 8. MCS with 10000 iterations are also tested to verify the accuracy shown in Fig. 8. The weight, mean vector and covariance matrix of line power obtained by the proposed method are shown in table IV.

TABLE IV. PARAMETERS OF TWO LINES POWER

$\omega_1=0.5571, \mu_1=\begin{bmatrix}2.3302\\5.1516\end{bmatrix}, \omega_2=0.4429, \mu_2=\begin{bmatrix}2.4551\\5.0964\end{bmatrix}$
$\Sigma_1=\begin{bmatrix}0.0071 & -0.0027\\ -0.0027 & 0.0012\end{bmatrix}, \Sigma_2=\begin{bmatrix}0.0087 & -0.0028\\ -0.0028 & 0.0009\end{bmatrix}$

In Fig. 8, the X-axis and Y-axis are the active power on the two transmission lines respectively, and the height of the six figures is shown by the right color column, representing the JPDF and JCDF of the active power on two transmission lines. The yellow area is maximum area of JPDF and JCDF, followed by green and blue. The probability distribution of the active power on multiple transmission lines can be determined simultaneously from the JCDF and JPDF. The ARMS and MAE of two methods compared with MCS are shown in table V, which further verifies the higher accuracy of the proposed method.

TABLE V. ERROR ANALYSIS OF TWO METHODS IN 39-BUS SYSTEM

	ARMS	MAE
GA based PDF	0.0039	0.1795
GA based CDF	1.87×10^{-5}	0.0169
EM based PDF	0.0065	0.7198
EM based CDF	0.0069	0.2136

The above is the analysis of JPDF and JCDF for several line power. Then, the probabilistic distribution of voltage amplitude of the modified IEEE 39 bus system is analyzed in following section. The location and output power of the wind farms are the same as above. To demonstrate the results of the proposed method, bus 27 is selected randomly to solve its PDF and CDF by MCS with 10000 iterations, proposed method and EM based method respectively. Fig. 9 is PDF and CDF of voltage amplitude of bus 27.

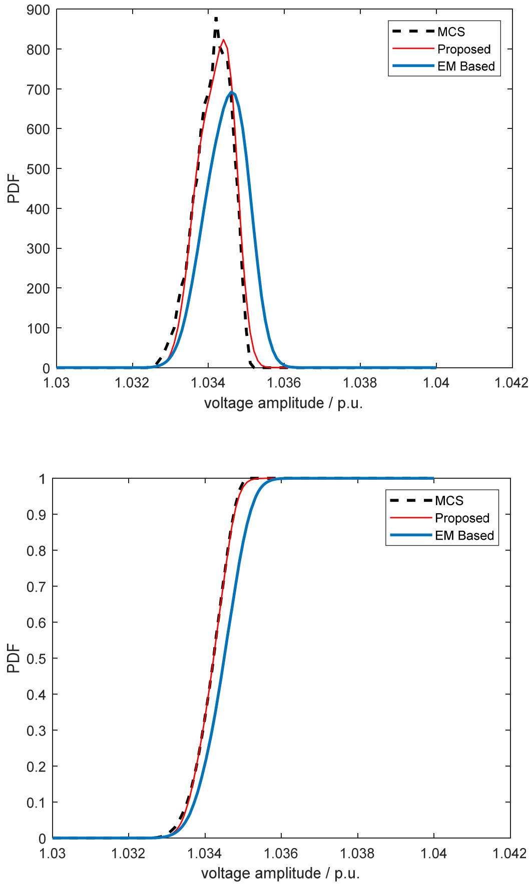

Figure 9. PDF and CDF of voltage amplitude in modified 39 bus system

As is shown in Fig. 9, red lines represent the PDF and CDF obtained by the proposed method; blue lines are PDF and CDF of voltage amplitude obtained by EM; black lines are obtained by MCS. Intuitively, the proposed method matches closely with MCS compared with method based on EM. The ARMSs of the proposed method and the method based on EM are 0.00035 and 0.031 respectively, and the MAE of the proposed method and the method based on EM are 0.022 and 0.211. From the Fig. 9, the voltage fluctuation is extremely small, which indicates that wind power fluctuations of wind farms do not affect the wide range of voltage fluctuations.

C. *Modified IEEE 118-bus system*

In this section, a modified IEEE 118-bus power system with 7 wind farms is used to further demonstrate the feasibility and effectiveness of the proposed method, as is shown in Fig. 10. In the system, synchronous generators connected to bus 10, 59, 61, 65, 66, 80, 89 are all replaced by wind farms. Since there are no available wind farms output data, in order not to lose generality and further verify the feasibility of the proposed method, the 7 wind farms outputs are all assumed to obey Weibull distribution [23-24]. Other detailed parameters of the power system are shown in [25]. The GMMs with 4 components which are modeled by GA and EM respectively are used to model the 7 wind farms outputs. In order to visualize the results obtained by MCS, proposed method and EM based method respectively. Voltage amplitude of bus 47 is selected to obtain their PDF and CDF shown in Fig. 11, and then active power on lines between buses 45 and 49 and lines between buses 2 and 1 are randomly selected to determine their JPDF and JCDF, as is shown in Fig. 12. Besides, in order to further demonstrate the effectiveness of the proposed method, active power on three lines, four lines and six lines are selected randomly to determine their JCDF, and the AMSE and MAE compared with MCS are shown in table VI.

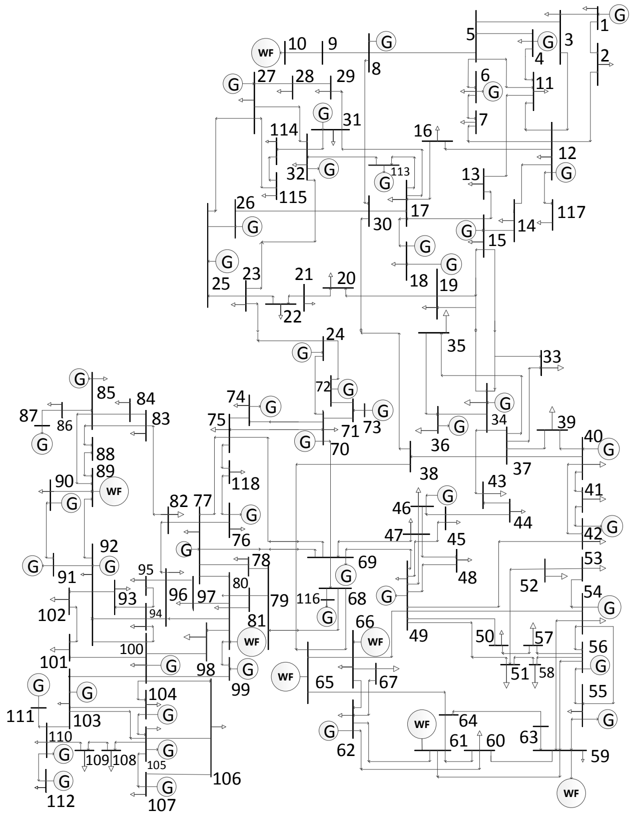

Figure 10. Modified IEEE 118-bus system

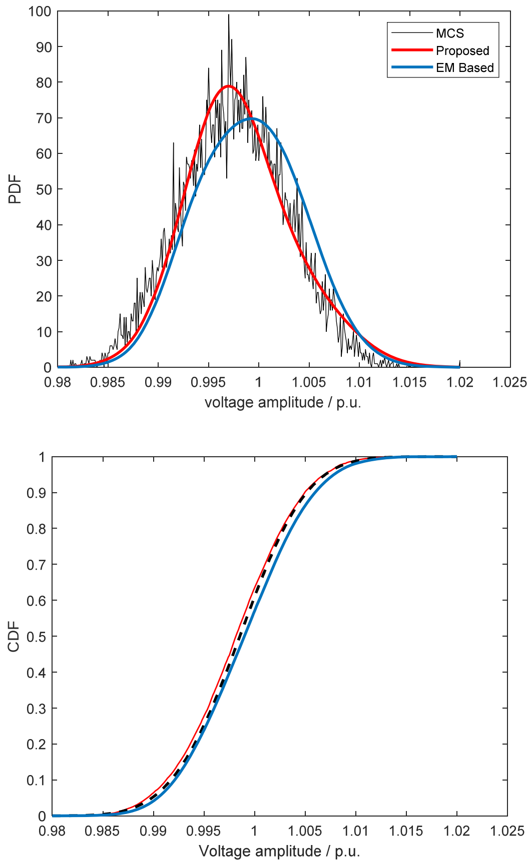

Figure 11. PDF and CDF of voltage amplitude in modified 118 bus system

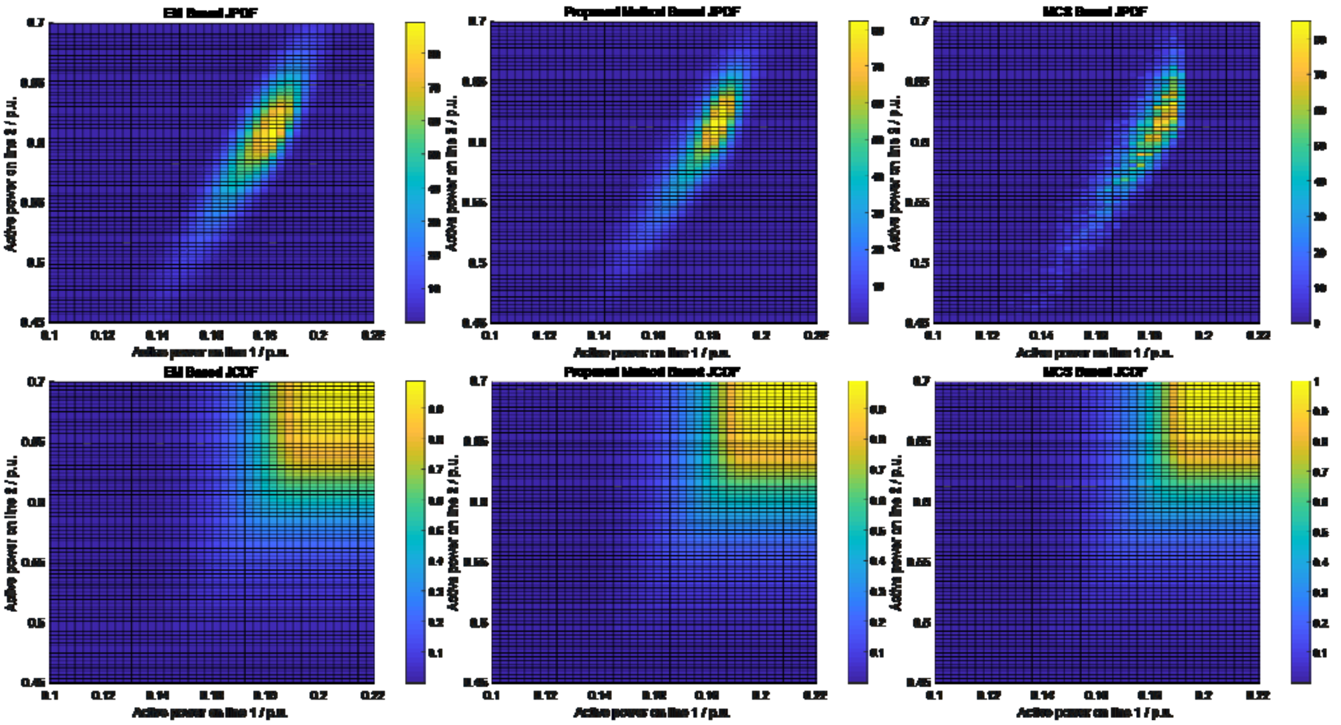

Figure 12. JCDF and JPDF of two lines in modified 118-bus system with seven wind farms

TABLE VI. ERROR ANALYSIS OF TWO METHODS IN 118-BUS SYSTEM

	ARMS of GA	MAE of GA	ARMS of EM	MAE of EM
2 lines	1.85×10^{-5}	0.0067	0.0381	0.1371
3 lines	2.51×10^{-5}	0.0048	0.0276	0.2568
4 lines	3.62×10^{-5}	0.0059	0.0565	0.5443
6 lines	2.63×10^{-5}	0.0068	0.0337	0.4358

In Fig. 12, the PDF and CDF of active power obtained by the proposed method match very closely with MCS, but the method based on EM gives deviations. The MAE and AMSE of the proposed method compared with MCS are 0.087 and 0.0027. The MAE and AMSE of the EM based method compared with MCS are 0.218 and 0.0289. Meanwhile, It can be concluded form the table VI that the ARMS and MAE obtained by the proposed method in modified 118 bus system with 7 wind farms are still tiny and smaller than EM based methods, which verifies the feasibility of the proposed method.

The calculation time of the proposed method are shown in table VII. MCS takes much more calculation time than the proposed method in the same system, and the calculation time of MCS increases greatly with the number of buses increasing. The proposed method in this paper meets the accuracy requirements completely and improves the accuracy of EM based method. More importantly, the proposed method in this paper performs less time than MCS, and calculation time does not increase greatly with the number of buses increasing.

TABLE VII. COMPARISONS OF CALCULATION TIME

	Modified 39-bus system	**Modified 118-bus system**
MCS	52 (s)	620 (s)
Proposed	0.326 (s)	0.462 (s)

V. CONCLUSION

The following are the main contributions of the proposed method based on improved GMM:

(1) The proposed method is less affected by the initial value, and can attain global optimality more easily, thereby improving the accuracy of GMM.

(2) Probabilistic load flow can be quickly solved by analytical method based on improved GMM, and its accuracy meets MCS studies.

(3) Joint distribution obtained by the proposed method can provide more comprehensive and exact assessment for the probability of multiple transmission lines being overloaded simultaneously.

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