Generalized Model of Economic Dispatch Optimization as an Educational Tool for Management of Energy Systems

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Abstract—**The available literature about Economic Dispatch ED problems is rich. In most of the cases the readers need specialized multidisciplinary knowledge of control systems, advanced optimization techniques, computers software and energy systems. Integration of ED problems in syllabus of economic management of energy systems requires knowledge of control engineering, and economics. There is a gap of knowledge between classical control theory most of graduates possess, and necessary advanced optimal control techniques for working in the field of ED problems. This article presents an educational generalized method EGM_ED for modeling, and solving the optimization of ED problems of thermal energy systems, using matrix mathematics, and matlab software. The solution is deducted mathematically, computed with matrices, and do not uses solvers from toolboxes. Can be applied in many different situations of energy systems with big numbers of generators. EGM_ED method is easier accessed by students, early carrier engineers, and practitioners. Using our matrix-based modelling for solving ED problems will offer access to students, and to other interested people to better learn, specialize and take critical managerial decisions in related energy industry jobs.**

Index Terms—**engineering education, energy management, economics, power generation dispatch, algorithms.**

I. Introduction

Economic management of energy systems aims at efficiency and savings, and rely on capital investment, and costs. The objectives are: to determine the actual power output of each of the available generating units that is needed to supply the electric load demand, and to comply with the technical and security constraints, and with the limits imposed by the transmission network. This problem, designated as Economic Dispatch (ED) is essential for the economic management of energy systems, and is considered in all electricity markets.

ED problems concern the optimization of power generation by installed units, to minimize the total fuel cost, subject to transmission, and operational constraints, and load demand. ED solutions must manage the distribution between electric generation units, with constraints, and minimize the total operational cost, [1-3].

The ED problem becomes complex because of the big number of generators, and constraints, combined in linear, or nonlinear objective functions, and connected to hybrid grids. Thus, optimizing the assignment of which generating units should produce electricity, minimizing the total cost, while studying the design for low-cost electricity production in the future power grids, receives renewed attention, [4-6].

From Educational Perspective, recent research shows the need for updated curricula, mainly because engineering education in ED as multi-disciplinary field is essential to support the continuing growth of energy economics. The engineering education of economics, and energy efficiency of power plants is important, and must modernize continuously, to prepare the graduates for existent, and new enterprises, and investments in energy markets, [7].

From Academic Perspective, at undergraduate and graduate level, teaching management of energy systems needs a blend of engineering background with analytical methods, computer-based solutions in optimal control, economics, and multiple criteria assessments, [8].

One reported multidisciplinary undergraduate course addresses gaps in energy education, and provides skills for engineering needs [9]. Course includes lectures, laboratories, and projects in design of hybrid power system for economic integration. The assessment of students in engineering education applies internationally adopted principles, [10-12].

Optimization of ED has been expanding during decades. Efficient theoretical, and practical techniques have been developed. Many uses Lagrange multipliers theory. Practical applications need working algorithms, and models, that can be easily used by students, and practitioners.

Existent classical software tools, and programs for ED do not run easily in new computers, and operating systems and programs. They can handle only a limited number of generators.

On the other hand, custom software used by energy companies is not widely available, nor can be accessed by academics for educational purposes. Their cost is too expensive for university budgets. As a result, the introduction of new emerging technologies in every-day teaching is a slow process, [13].

It happens in textbooks, and publications that many authors present solved applications, with few generating units (from three up to five), and selected characteristics of cost functions, which converge easily. Thus, the student, or the non-expert, receives the wrong information, that ED problems are easy to solve, and always have good solutions.

Other publications begin with some equations from literature, add analytical formulations of other components, then report numerical results, but without giving details about how to solve the complete problem. Instead of receiving detailed solutions, the readers are directed to

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search in other references of other authors, which also wrote their articles based on other references of other authors, and so on. Sometimes, from successions of articles, the measurement units do not match, or the variables have different currencies, or the currencies used are not consistent to the timing of carrying out the research.

Many publications refer theoretical study cases, with simulation, which uses tool-boxes from commercially available software, where somebody has to input some coefficients and data, and, after some tests, decide which is a good solution.

From educational point of view, unfortunately, such category of publications cannot be used by students, or researchers, in a learning process.

We carried out the investigation, and analysis of selected methods, and techniques for the optimization of ED in thermal power systems. We concluded that the existent literature about ED is vast, and requires specialized knowledge of optimal control, mathematics, and computer software. Thus, overrides the economics' aspect which is the main target. Not every reader possesses such multidisciplinary knowledge and, consequently, cannot easily understand, nor apply, such a huge amount of specialized information.

For all above reasons we decided to develop the new Educational Generalized Model, for the ED optimization, EGM_ED, with the associated software, which can be used as an educational tool for teaching economic management of energy systems to students, and engineers. The benefits of developing this tool include comprehension of energy system economics problem, learning how to formulate, and solve them, and how to interpret the solutions using problem-solving methodology in engineering.

To address this issue, we developed our own educational generalized method for solving optimization of ED problem EGM_ED, and designed a new model for ED based on matrix theory mathematics. The algorithm, and software are programed with matlab and produced a complete routine.

Students will learn economic management of energy systems in academic environment, using the model and software EGM_ED, and applications for particular study cases. Students' involvement with real study cases will improve their professional competencies for postgraduate studies, research projects, and future jobs.

Our work advances prior knowledge with a new model, and solution and comparison with other existent. Our purpose is to offer access to scientists, working in technical, and economic fields, to easier solve ED problems, by using our EGM_ED model for big numbers of generators, developed in compact form, and requiring knowledge of mathematics of matrices, and Matlab.

II. SELECTED METHODS FOR SOLVING ED PROBLEMS

The ED problem has been studied during decades searching for solutions that would lead to higher savings in operating cost, [2-3], [14].

A generic software for ED problems estimates the optimal value of power to be generated with the least possible fuel cost using modified Lambda-Iteration method, based on the assumption of equal incremental cost, [15]. In [16], the authors examine the conditions that affect the optimality of this problem, and propose the interior-point method based linear programming.

Publication [17] address the ED problem using Lambda-iteration method, and considering prohibited operating zones due to physical operational limitations of power plant. Other algorithms use Lagrangian multipliers with monotonic cost functions, [18]. An inaccurate ED solution results if the input-output curve is nonlinear, non-smooth, and non-convex, due to the effect of valve-point effect, [19-20].

ED involving Combined Cycle CC units is a non-convex optimization problem because CC units have multiple operating configurations depending on the number, and status of combustion, and steam turbines. The state space model of CC units for dynamic programming, and Lagrangian relaxation to security constrained short-term scheduling is used in [21]. Using the calculation of infimal convolution, the authors of [22] find a global solution for ED. In [23] is proposed a model for scheduling the CC gas units by mixed-integer programming. The modeling of modes with combustion, and steam turbines, require approximations in sub-optimal schedules for fuel input, power output, which can save operating costs.

The merit order loading ranks on ascending order the prices of available sources of energy, and the amount of energy that is generated, [24]. The ranking is such that those units with the lowest marginal costs are the first ones to be brought online to meet demand, and the units with the highest marginal costs are the last to be brought online. ED in this way lowers the cost of electricity. Sometimes generating units must be started out of merit order, due to transmission congestion, system reliability, or other reasons. A merit order reduced gradient algorithm for on-line generation control with linear programming, and merit order loading, was used with monotonically increasing incremental heat rate curve, [24]. A merit order loading method with linear decreasing found application in [25].

A consensus for parallel, and distributed algorithm using Lagrange multipliers for ED is presented in [26]. In [27] is proposed a mixed integer quadratic programming for dynamic ED. With piecewise linearized non-linear-non-smooth cost, if solved in a single step, the optimization suffers convergence stagnancy, while with multi-step, breaks the convergence stagnancy.

A coordinated dynamic ED with integrated large-scale distributed energy resources, proposes a decentralized method to solve the problem using multi-parametric quadratic programming with boundary variables exchanged between networks. The method can achieve a global optimal solution, and has economic benefits compared to the isolated ED method, [28].

Lagrangian relaxation method splits the large-scale optimization with coupled structure into several smaller sub-problems. The augmented Lagrangian relaxation method has an added penalty term in an augmented Lagrangian function, which makes difficult the full decomposition. A diagonal quadratic approximation method produces an approximated block separation of the penalty term, so that the ED model is decomposed into several single-period ED models that are solved in parallel, [29]. For large-scale integration of renewables, ED is considered a two-stage dual problem via Lagrangian relaxation, with updating the multiplier, [30].

The joint ED optimization, with slow and fast timescale frequency regulation, is approaching the fast timescale sub-problem using a distributed frequency control algorithm that preserves network stability during transients, and the slow timescale using an efficient market mechanism, [31].

Genetic Algorithm GA, Evolutionary Programming EP, Particle Swarm PS, Differential Evolution DE, Grey Wolf GW, and some other, are evolutionary algorithms with searching mechanisms per generation, and have good performances for ED problems, [18], [32].

The authors in [33] use a GA method to solve ED with non-convex cost curve, and valve point effects. The algorithm utilizes information of perspective solutions to evaluate optimality, and computation to increase program efficiency, and accuracy, such as mutation prediction, selectiveness, interval approximation, and penalty factors.

The stochastic property makes EP algorithms appropriate for non-convex optimization of ED, [34]. In [34] an EP algorithm with non-smooth fuel cost functions, determines the global, or near global optimal solutions, as an alternative to Lagrangian based algorithms.

Usually, during optimization is used entire capacity of power plant, distributing the demand for all generation units, including the least efficient ones. In [35] is presented an GA optimization including turning off the generators with higher losses. The incremental cost of fuel is used to determine the best parameters of active power of each generating unit, ensuring that the demand, and total losses are equal to the total generated power, minimizing the total cost of fuel.

Surveys about solutions of ED problem through PS optimization are in [36-37]. Publication [38] presents a PS optimization to solve ED with multiple fuels, with non-smooth cost functions, and constraints.

In [39], the authors implement GA, EP, PS algorithms for ED problems, and compare with mixed integer linear programming. The trajectory, and searching path of each algorithm show that the stochastic optimization techniques provide approximate global optimal solution for non-convex optimization problem. Other selected publications with PS optimization are [40-44].

EP and PS optimization techniques are effective for small, and medium-sized power systems. The hybrid hierarchical evolution algorithm developed for solving non-convex, multi-zone ED for large systems, integrates the capabilities of PS and DE optimization, to improve the search efficiency, [45]. A hybrid swarm algorithm combining self-assembly and PS optimization, considers integration of components to balance exploration, and exploitation and improve the process, thus obtains improved speed for convergence, [46]. In [47-48] is presented a GW optimization algorithm, which can contribute to solve ED problems.

III. Formulation of Optimization of ED Problem

The operation of power systems involves the management of multiple generating units, that are used to supply the electric load demand. This management requires considering specific technical aspects of generating units such as power-output limits, and network constraints of the power system.

Besides the technical aspects needed to generate a reliable supply of energy, it is also necessary to consider an efficient economic management.

The ED problem determines the actual power output of each generating unit that is needed:

- to supply all electric load demands of the network;
- minimizing total operating costs;
- meeting technical and security constraints, and
- complying with the constraints of the transmission network.

We consider a number of generating units that are used to supply the electric load demand in a given power system. These generating units are indexed by *i*, $i = 1, \ldots, N$, where N is the total number of electricity generating units.

The objective of the ED problem is to determine the production of every generating unit *i* in the given power system where is needed:

- to minimize the total costs;
- to supply the electric load demand, and
- to meet the technical and security constraints, [1-3].

The following describe the *main components* of the ED problem, including the economic, technical, and security constraints of generating units, that must be satisfied.

Costs of Generating Units, or the cost of producing electricity by thermal generating units can be expressed as:

$$C_i = C_{F,\mathrm{i}} + C_{V,i} \quad \forall i \tag{1.1}$$

where:

C_i is the total cost of generating unit *i*,

$C_{F,\mathrm{i}}$ is the fixed cost, or no-load cost of generating unit *i*,

$C_{V,i}$ is the variable cost of generating unit *i*.

When a generating unit produces electricity, it has a variable cost $C_{V,i}$ that can be expressed as a function of the output power P_i :

$$C_{V,i} = C_{Vi} \cdot F_i(P_i), \quad \forall i \tag{1.2}$$

where: C_{Vi} is the coefficient of variable cost of generating unit *i*, and P_i is the output power of generating unit *i*. The fixed, and variable costs constitute the running costs of generating units, incurred by producing electricity.

As an alternative to total cost, is introduced the cost rate:

$$F_i(P_i) = C_i / t \quad \forall i \tag{1.3}$$

where: *t* is the time interval, usually considered 1 hour, *t=1h*.

The running generating units must supply the electric power demand. Essential constraints in the operation of power system are that the sum of all output powers P_i equals the total system electric load demand P_L, and thus, sets to zero function Φ :

$$\Phi = P_L - \sum_{i=1}^{N} P_i = 0 \tag{2.1}$$

For security reasons, the total output power available online should be larger than the actual load demand by a prespecified amount. This is formulated as:

$$\sum_{i=1}^{N} P_{i,\max} \geq P_L + R_L \tag{2.2}$$

where: R_L is the amount of required reserve, or the capacity available over the total electric load demand P_L .

Thermal generating units cannot operate below a minimum power output, and above a maximum power output. These *technical constraints* can be expressed as $2 \cdot N$ inequality constraints: the power output P_i must be greater than, or equal to, the minimum power permitted $P_{i,\min}$, and, less than, or equal to, the maximum power permitted $P_{i,\max}$ of generating unit *i*:

$$P_{i,\,min} \le P_i \le P_{i,max}\,, \quad \forall i \tag{2.3}$$

where:

$P_{i,\,min}$ =generation lower limit, minimum output of *i-th* unit;

$P_{i,max}$ =generation upper limit, maximum output of *i-th* unit.

Equation (2.3) is *the power bound* of each generating unit *i.* The left-hand side of constraints imposes that if generating unit *i* is online, its power output should be above the minimum power output. Respectively, the right-hand side of constraints imposes that if generating unit *i* is online, its power output should be below the maximum power output.

Thus, the ED problem is formulated as an optimization problem of an objective function Min_F_T. The objective function F_T is the sum of cost rates of all generating units *i,* $F_i(P_i)$, [1-3]:

$$F_T = F_1(P_1) + F_2(P_2) + \cdots + F_N(P_N) = \sum_{i=1}^{N} F_i(P_i) \tag{3}$$

subject to constraints (2.1)- (2.3), where:

F_T = total cost rate of the system of *N* generating units;

$F_i(P_i)$ = fuel cost rate function of *i-th* generating unit;

P_i = electric power generated by *i-th* generating unit.

The objective function (3) is the operating cost rate. Constraint (2.1) defines the *power balance,* and constraint (2.3) imposes *power bounds* on generating units.

We consider ED problem for an energy system of *N* thermal generating units, serving a specific electric load demand P_L. The ED problem minimizes the objective function, which is the total cost rate function F_T, subject to the power balance, and power bound constraints.

The objective function F_T includes fixed costs (investment, personal, costs non-varying with the output), and variable cost (fuels, energy, maintenance material, taxes). General external costs, that arise from electric utilities activities are not considered where these do not fall on the utilities themselves, such as accidents, damages to property or health, environmental damage from air pollutants, discharges of waste heat, etc., [1-3].

The minimization of objective function F_T is approached using Lagrangian *L*, with constraints, and the method of solution uses Lagrange multipliers, [2-3], [49].

We formulate Lagrangian *L* by adding to the objective function F_T, (3), the constraint Φ, (2.1), multiplied by lambda λ:

$$L = F_T + \lambda \cdot \Phi \tag{4}$$

The optimal solution of the objective function F_T is obtained by equaling to zero all *i* first partial derivatives of Lagrangian *L* with respect to each one of the independent variables P_i, and with respect to lambda λ:

$$\frac{\delta L}{\delta P_i} = 0 \Rightarrow \frac{\delta F_i(P_i)}{\delta P_i} - \lambda = 0 \tag{5.1}$$

$$\frac{\delta L}{\delta \lambda} = 0 \Rightarrow \sum_{i=1}^{N} P_i - P_L = 0 \tag{5.2}$$

where:

$\frac{\delta F_i(P_i)}{\delta P_i}$ is the incremental cost rate of unit *i,* or the cost sensitivity. It is computed when one parameter changes while the others remain constant.

The condition for existence of a minimum cost operating point is that the incremental cost rates $\frac{\delta F_i(P_i)}{\delta P_i}$ of generated powers P_i become equal to λ. From (3), (5.1)-(5.2), the necessary conditions are (6.1)-(6.3), [3]:

$$\frac{\delta F_i(P_i)}{\delta P_i} = \lambda \quad \text{for} \quad P_{i,\,min} \le P_i \le P_{i,max} \tag{6.1}$$

$$\frac{\delta F_i(P_i)}{\delta P_i} \le \lambda \quad \text{for} \quad P_i = P_{i,max} \tag{6.2}$$

$$\frac{\delta F_i(P_i)}{\delta P_i} \ge \lambda \quad \text{for} \quad P_i = P_{i,\min} \tag{6.3}$$

IV. MODELING ED PROBLEM WITH MATRICES

To solve the above problem, we designed the new mathematical model based on matrix theory, and the algorithm and software EGM_ED. The definitions, analysis, and results are presented in the followings.

We introduce matrix formulation in (3), (2.1), (2.3), (4), (5.1)-(5.2), (6.1)-(6.3).

For each generating unit *i* we define the cost rate $F_i(P_i)$ as a product of heat rate $H(P_i)$ and fuel cost FC_i:

$$F(P_i) = H(P_i) \cdot FC_i \tag{7}$$

We define the vectors of generated powers $[P]$, cost rates $[F(P)]$, fuel costs $[FC]$ of dimensions *N*, and matrix of heat rates $[H(P)]$, in (8)-(12):

$$[P] = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} \tag{8}$$

$$[F(P)] = \begin{bmatrix} F_1(P_1) \\ F_2(P_2) \\ \vdots \\ F_N(P_{1N}) \end{bmatrix} \tag{9}$$

$$[FC] = \begin{bmatrix} FC_1 & FC_2 & \cdots & FC_N \end{bmatrix}^T \tag{10}$$

The characteristic functions of thermal units are defined as in [2-3]. For the heat rates $H_i(P_i)$, we use quadratic (convex) functions:

$$H_i(P_i) = H_{i,2} \cdot P_i^2 + H_{i,1} \cdot P_i + H_{i,0} = \sum_{j=0}^{2} H_{i,j} \cdot P_i^j \tag{11}$$

and $j = 0,1,2$. Equation (11) is written with matrices:

$$\left[H(P)\right]=\begin{bmatrix}H_{1,2}\\H_{2,2}\\\vdots\\H_{N,2}\end{bmatrix}_{\bullet}\cdot\left[P\right]^2_{\bullet}+\begin{bmatrix}H_{1,1}\\H_{2,1}\\\vdots\\H_{N,1}\end{bmatrix}_{\bullet}\cdot\left[P\right]+\begin{bmatrix}H_{1,0}\\H_{2,0}\\\vdots\\H_{N,0}\end{bmatrix} \quad (12)$$

where, matrixes with dots $\left[H\right]_{\bullet}\cdot\left[P\right]_{\bullet}$ denotes multiplication of element (i,j) of $\left[H(P)\right]$ by element i of $[P]$, $\left[P\right]^2_{\bullet}$ denotes the 2nd power of each element i of $[P]$, and $\left[FC\right]^T$ is the transposed matrix, (in Matlab syntax).

From (7)-(12), we obtain the cost rate matrix $\left[F\right]$:

$$\left[F\right]=\left[H\right]\cdot\left[FC\right] \quad (13)$$

Using matrix formulation (8)-(13), the system of equations (5.1)-(5.2) becomes:

$$\left[A\right]\cdot\left[P_0\right]+\left[B\right]=0 \quad (14)$$

where, matrix $\left[A\right]$ is of dimensions $(N+1)$-by-$(N+1)$ and vectors $\left[B\right]$ and $\left[P_0\right]$ are of dimensions $(N+1)$:

$$\left[A\right]=\begin{bmatrix}2\cdot H_{1,2}\cdot FC_1 & 0 & \cdots & 0 & -1\\ 0 & 2\cdot H_{2,2}\cdot FC_2 & \cdots & 0 & -1\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 2\cdot H_{N,2}\cdot FC_N & -1\\ -1 & -1 & \cdots & -1 & 0\end{bmatrix} \quad (15)$$

$$\left[B\right]=\begin{bmatrix}H_{1,1}\cdot FC_1\\ H_{2,1}\cdot FC_2\\ \vdots\\ H_{N,1}\cdot FC_N\\ P_L\end{bmatrix} \quad (16)$$

$$\left[P_0\right]=\begin{bmatrix}P_1\\P_2\\\vdots\\P_N\\\lambda\end{bmatrix} \quad (17)$$

The solution of (14) is $\left[P_0\right]$, in (18), and gives the initial, or the global optimal vector:

$$\left[P_0\right]=-\left[A\right]^{-1}\cdot\left[B\right] \quad (18)$$

From vector $\left[P_0\right]$ we obtain the optimal values for generated powers $P_1,P_2,\cdots,P_N$, and the Lagrange multiplier $\lambda=\lambda_1$. Vector $\left[P_0\right]$ is the optimal solution of the unconstrained system (2.1)-(3): minimizes the cost function F_T, from (3) and fulfil the power balance for the load demand from (2.1).

Depending on the value of load demand P_L, and fuel costs $\left[FC\right]$, we compute the solution $\left[P_0\right]$.

The value $\lambda=\lambda_1$ is considered as initial incremental cost rate, and $P_1,P_2,\cdots,P_N$ as initial operating point, for starting our λ-iteration procedure.

We introduce a small displacement, or a small variation $\pm\Delta\lambda$ from the initial λ_1, which can be in the range of $0.1\%\leq\Delta\lambda\leq 1\%$:

$$\lambda=\lambda_1\pm\Delta\lambda \quad (19)$$

According to (2.3), in all situations, the values of optimal solutions P_i must be bounded to the generation upper limits, and lower limits. To apply the boundary conditions to the initial set of values P_i from vector $\left[P_0\right]$, we must formulate in matrix form the constraints (6.1)-(6.3):

$$[P_{min}]=\begin{bmatrix}P_{1,\min}\\P_{2,\min}\\\vdots\\P_{N,\min}\end{bmatrix} \quad (20.1)$$

$$[\mathrm{P}_{max}]=\begin{bmatrix}P_{1,\max}\\P_{2,\max}\\\vdots\\P_{N,\max}\end{bmatrix} \quad (20.2)$$

$$\begin{bmatrix}P_{1,\min}\\P_{2,\min}\\\vdots\\P_{N,\min}\end{bmatrix}\leq\begin{bmatrix}P_1\\P_2\\\vdots\\P_N\end{bmatrix}\leq\begin{bmatrix}P_{1,\max}\\P_{2,\max}\\\vdots\\P_{N,\max}\end{bmatrix} \quad (20.3)$$

$$[P_{min}]\leq[P]\leq[\mathrm{P}_{max}] \quad (20.4)$$

From (6.1) - (6.3), we obtain the constraints in matrix formulation, in (21.1) - (21.3):

$$\left[\frac{\delta F_i(P_i)}{\delta P_i}\right]=\left[A_\lambda\right]\cdot\left[P_\lambda\right]+\left[B_\lambda\right]=0 \quad (21.1)$$

$$\left[\frac{\delta F_i(P_i)}{\delta P_i}\right]=\left[A_\lambda\right]\cdot\left[P_\lambda\right]+\left[B_\lambda\right]<0 \quad (21.2)$$

$$\left[\frac{\delta F_i(P_i)}{\delta P_i}\right]=\left[A_\lambda\right]\cdot\left[P_\lambda\right]+\left[B_\lambda\right]>0 \quad (21.3)$$

where: $\left[A_\lambda\right]$ is a diagonal matrix N-by-N, vectors $\left[L_\lambda\right]$ and $\left[B_\lambda\right]$ are of dimensions N, and $[B(1:N)]$ are the first N elements of $\left[B\right]$ (in Matlab syntax), in (22)-(24):

$$\left[A_\lambda\right]=\begin{pmatrix}2\cdot H_{1,2}\cdot FC_1 & & 0\\ & \ddots & \\ 0 & & 2\cdot H_{N,2}\cdot FC_N\end{pmatrix} \quad (22)$$

We generate

$$\left[L_\lambda\right]=\begin{bmatrix}\lambda\\\lambda\\\vdots\\\lambda\end{bmatrix} \quad (23)$$

$$[B_\lambda]=[L_\lambda]-[B(1:N)] \quad (24)$$

Equations (21.1)-(21.3) verify whether the solution is bounded by the upper limits $[\mathrm{P}_{max}]$, and lower limits $[P_{min}]$, and set the solution to the corresponding constraints. If the solution $\left[L_\lambda\right]$ verifies constraints, then this vector gives the *incremental cost vector* for all N units. The solution of (21.1)-(21.3) is vector $\left[P_\lambda\right]$:

$$[P_\lambda] = \begin{bmatrix} P_{1\lambda} \\ P_{2\lambda} \\ \vdots \\ P_{N\lambda} \end{bmatrix} \tag{25}$$

$$[P_\lambda] = [A_\lambda]^{-1} \cdot [B_\lambda] \tag{26}$$

Using the computed vectors $[L_\lambda]$ and $[P_\lambda]$ we verify (21.2)-(21.3). Also, we verify (27), where $\Delta\Phi \le 1\% \cdot \mathrm{P}_L$ and $\varepsilon \le 1\% \cdot \mathrm{P}_L$ are considered acceptable errors.

$$\Delta\Phi = \sum_{1}^{N} P_{i\lambda} - P_L < \varepsilon \tag{27}$$

The corresponding operational costs of generating units i are computed in vector $\mathrm{F}([\mathrm{P}_\lambda])$, (28):

$$\mathrm{F}([\mathrm{P}_\lambda]) = [\mathrm{H}([\mathrm{P}_\lambda])] \cdot [FC] \tag{28}$$

The total minimal operational cost F_λ for the generation of total load power is (29):

$$\mathrm{F}_\lambda = \sum_{i=1}^{N} \mathrm{F}([\mathrm{P}_\lambda]) \tag{29}$$

The incremental costs $\left[\frac{\delta F_i(P_i)}{\delta P_i}\right]$ for final $[L_\lambda]$ and $[P_\lambda]$ are computed from (21.1), and (25).

V. Optimization Algorithm

The algorithm for the EGM_ED problem, with small displacements of λ, (7)-(29), is presented below. The program is written in Matlab.

Step 0. Initialization.
Input: data of generating units.
Input parameters: fuel costs, load demand and allowed errors.
Step 1. Generate vectors $[FC]$, $[\mathrm{P}_{max}]$, $[P_{min}]$, (10), (20.1)-(20.2) and matrix $[H]$, (12).
Generate matrix $[A]$ and vector $[B]$, (15), (16).
Step 2. Compute vectors $[P_0]$, $[P]$, and λ_1, (18), (8), (17).
Step 3. Set $\lambda = \lambda_1 \pm \Delta\lambda$, (19), and start iterations.
Step 4. Compute $[P_\lambda]$, (25)-(26).
Step 5. Verify (21.1)-(21.3).
Step 6. Compute Φ, (2.1), and Verify (27).
Step 7. If *True*, Compute costs of generating units $\mathrm{F}([\mathrm{P}_\lambda])$, (28), and total minimal cost F_λ, (29), then *End*.
If *False,* Increment λ $\pm\Delta\lambda$ and go to Step 4.
End.

The data and parameters of the optimization algorithm for a given period of time are:

1) *Fixed input data of generating units:*
   - Number of generating units;
   - Characteristics of heat rates;
   - Prices of fuels.
   - Power bounds $[\mathrm{P}_{max}]$ and $[P_{min}]$.
2) *Parameters for running the optimization algorithm:*
   - Load demand;
   - Power balance.
3) *Parameters for accuracy of results:*
   - Size of displacement $\Delta\lambda$;
   - Size of error ε between total generated power and load demand.

The algorithm converges and stops when the computed error is smaller than the allowed ε, (27). This can occur in the two situations from (19): increasing $\lambda = \lambda_1 + \Delta\lambda$ or decreasing $\lambda = \lambda_1 - \Delta\lambda$. We can choose first to decrease λ by $\Delta\lambda$. If the algorithm converges at a small ε, then the iterations stop. If there is no convergence, then the iterations continue by increasing λ by $\Delta\lambda$, until attaining the small ε. Then the iterations stop and the result is the minimal total cost.

For obtaining a reliable comparison between two algorithms: the EGM_ED and software from [3], we used exactly the same parameters: the fixed input data of generating units, and the parameter for running the optimization algorithm which is the load demand at 3000MW, as presented in the following Study Case 1. We obtained very close results from this comparison.

Nevertheless, if we use other sets of parameters and data of other generating units, then the results will change, but these will belong to different study cases.

This quality is very useful from the Educational Perspective the EGM_ED model and software, because our students and researchers will tackle a diversity of study cases. This is particularly important because they will become users of the complete software.

VI. Study Case 1: Sixteen Generating Units from the Greek Energy System

In this study case we considered the power system with 16 generating units, constant network transmission losses at a penalty factor 7,15%, and the total load demand is $P_L = 3000MW$, or $78,43\% \cdot \sum_{i=1}^{N} P_{i,\max}$.

In Table A1, Appendix A, are the data of sixteen thermal generating units of the Greek power system, *Unit1-Unit16*, the generators' upper limits, lower limits, the heat consumption rates, the kind, and costs of fuels. There are considered two different fuels, lignite and full heavy oil. For each generating unit i the consumption of heat/hour $H_i(P_i)$ is a 2nd degree polynomial with coefficients $H_{i,j}$, (11), (12). The elements of matrix of cost rates $[F]$ are coefficients $F_{i,j}$, computed according to (13).

The cost rate functions of 16 generating units versus generated power are plotted in 3-D mesh style in Fig. 1.

In cases with big number of generating units, the 2-D plot do not show all details because many operating points of different generating units are superposed. In such situations, the 3-D mesh shows clearly all characteristics.

First, we compute the unconstrained optimal solutions of the power system and obtain the initial value of incremental costs $\lambda_1 = 12,7435$. Then, we solve (21.1)-(21.3) for small displacements of λ, from $\lambda_1 = 12,7435$ to $\lambda_\lambda = 11,2335$. Next, we compute the total generated powers $\sum_{i=1}^{N} P_{i,\lambda}$ and verify (27). The algorithm converges with an error of load demand $\varepsilon = 0.25\% \cdot P_L$.

The total generated power $\sum_{i=1}^{N} P_{i,\lambda}$, the load demand P_L, and the load not served, versus λ are plotted in Fig. 2. The convergence of algorithm, starting from the unconstrained optimal solution, up to the constrained optimal solution, is shown in Fig. 2. The total generated power converges to the load demand.

There are small final errors, and small amounts of load not served, at $\lambda_\lambda = 11,2335$. The number of iterations for convergence depends on load demand, and allowed errors.

In Fig. 3 are plotted the total operational costs F_λ, and the convergence of costs to minimal total operational cost $F_\lambda = 48.860,86€/h$, for small displacements of λ, from $\lambda_1 = 12,7435$ to $\lambda_\lambda = 11,2335$, (see also summary of results and solution in Table I). In Fig. 2, and Fig. 3 is clearly showing the effect of small displacements of λ, on the total generated power, on the load not served, on the total operational costs of power system, and the convergence of EGM_ED algorithm to the minimal total operational cost.

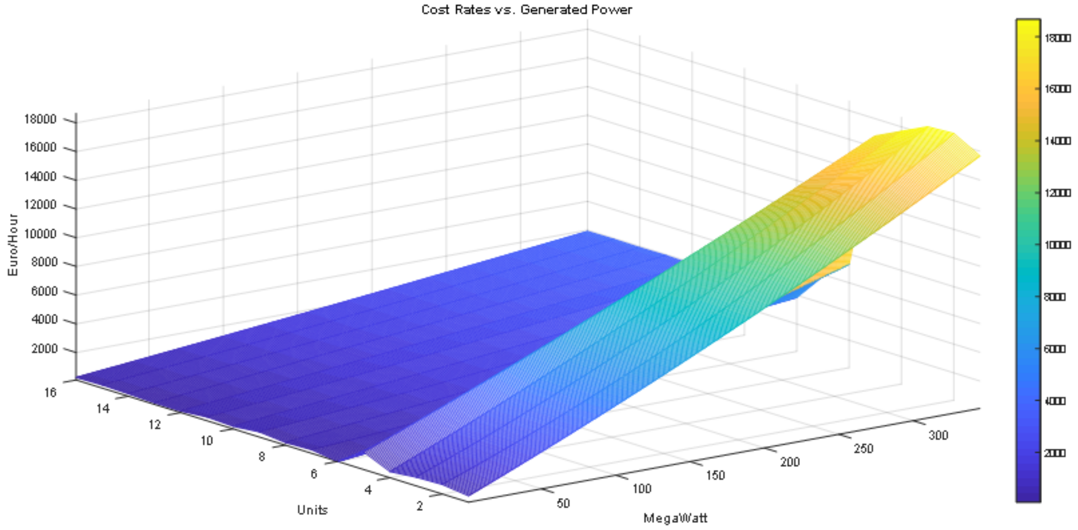

Figure 1. The cost rate functions of 16 generating units versus generated power MW, in 3-D mesh style

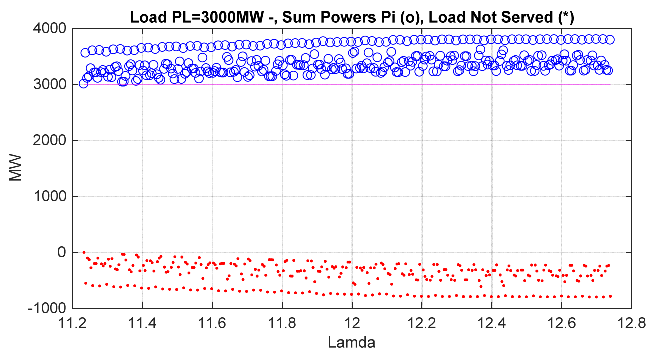

Figure 2. Convergence of EGM_ED from unconstrained optimal solution to constrained optimal solution. Small displacements of λ are from λ_1=12,7435 to λ_λ=11,2335. Total generated powers (blue o), load demand P_L=3000MW (magenta _), load not served (red .)

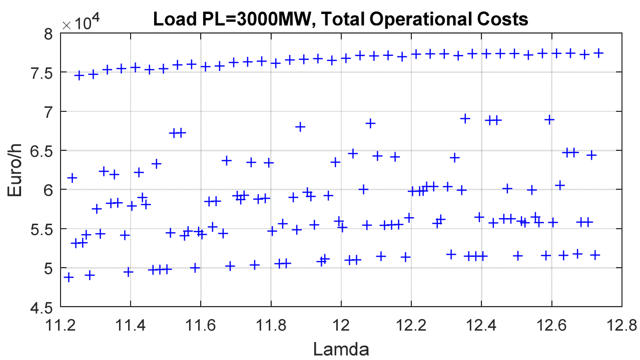

Figure 3. Convergence of costs to the minimal total operational cost F_λ=48.860,86€/h, (+ blue), from λ_1=12,7435 to λ_λ=11,2335

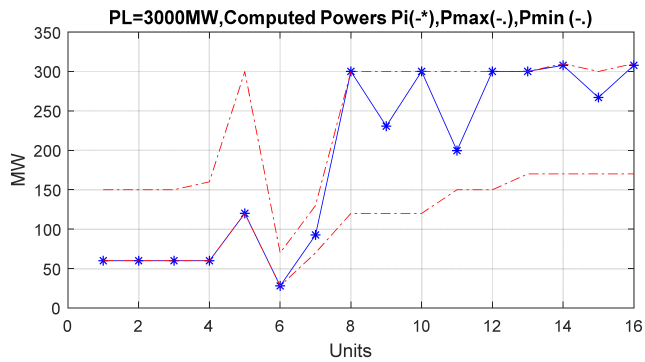

Figure 4. Generated powers [P_λ] (-*blue), and power bounds [P_{min}], [P_{max}], (-.red), of 16 generating units, at λ_λ=11,2335

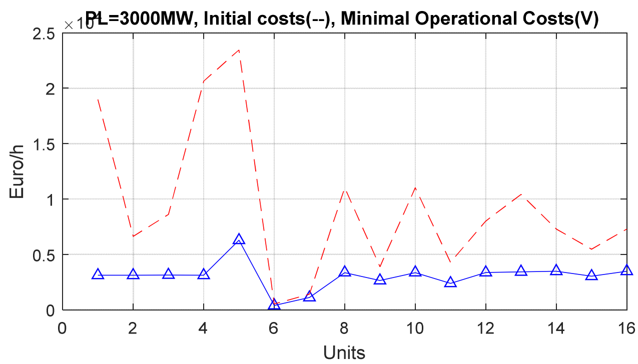

Figure 5. Operational costs of generating units: initial costs (-.red), and optimal contributing to total minimal operational cost, (-Δ blue), at λ_λ=11,2335

VII. STUDY CASE 1. ECONOMIC ANALYSIS

The generated powers $[P_\lambda]$, and the power bound constraints $[P_{min}]$, $[P_{max}]$ are plotted in Fig. 4. The corresponding operational costs for each generating unit contributing to total minimal operational costs are in Fig. 5.

For comparison reasons, the solution for the same ED problem, with the same 16 generating units, and heat consumption characteristics, was computed with the program in Turbo Pascal under dos operating system, [3]. This program [3] available from literature, is an educational software for solving ED problems using λ-iteration method, which can receive data of up to 20 generating units only.

In Table I are shown the results computed using the EGM_ED software, and the values computed using the software in Turbo Pascal, [3], which are marked with subscript ref. Specifically, for all generating units and load demand $P_L = 3000MW$, from EGM_ED software are the columns titled "Generated Power (MW) P_i", "Incremental Costs $\frac{\delta F_i(P_i)}{\delta P_i}$", and "Operational Costs F_i".

The values computed with the software from [3] are in the columns titled "Generated Power (MW) $P_{i,ref}$", "Incremental Costs $\left.\frac{\delta F_i(P_i)}{\delta P_i}\right|_{,ref}$", and "Operational Costs $F_{i,ref}$". The two sets of results are compared and are very close.

The results in Fig. 4, and Table I, show that generating units 1, 2, 3, 4, 5, 6 function at their minimum power $P_{\min}$, generating units 8, 10, 12, 13 function at their maximum power P_{max}, while generating units 7, 9, 11, 14, 15, 16 function at levels between $P_{\min}$, and P_{max}.

In Fig. 6 is shown the optimal trajectory of cost rates on the 3-D surface from Fig. 1, for the 16 generating units and the Megawatt generated.

The Mean Minimal Cost of all generating units (with all fuels) at €3.053,80, is higher than the Mean Minimal Cost of generating units with fuel 2 at €2.736,23 and lower than the Mean Minimal Cost of generating units with fuel 1 at €3.752,47, in Fig. 7 and Table II.

TABLE I. GENERATED POWER, INCREMENTAL COSTS, OPERATIONAL COSTS. COMPARISON TO VALUES OBTAINED WITH SOFTWARE [3]

Generating Units		**Generated Power (MW)**			**Incremental Costs (€/MWh)**		**Operational Costs (€/h)**	
		P_i	$P_{i,ref}$	**Boundary Values**	$\frac{\delta F_i(P_i)}{\delta P_i}$	$\left.\frac{\delta F_i(P_i)}{\delta P_i}\right\|_{,ref}$	F_i	$F_{i,ref}$
Fuel 1. Heavy Full Oil	1	60,00	60,00	$P_i = P_{i,\min}$	45,0645	45,0645	3.112,73	3.112,73
	2	60,00	60,00	$P_i = P_{i,\min}$	42,5289	42,5289	3.114,46	3.114,46
	3	60,00	60,00	$P_i = P_{i,\min}$	43,5874	43,5874	3.139,52	3.139,52
	4	60,00	60,00	$P_i = P_{i,\min}$	45,4112	45,4112	3.117,88	3.117,88
	5	120,00	120,00	$P_i = P_{i,\min}$	43,4761	43,4761	6.277,75	6.277,75
Fuel 2. Lignite	6	28,00	28,00	$P_i = P_{i,\min}$	12,0857	12,2323	391,20	391,20
	7	92,89	*92,90*	$P_{i,\,min} \le P_i \le P_{i,max}$	11,2335	11,2323	1.112,00	1.111,78
	8	300,00	300,00	$P_i = P_{i,\min}$	10,7590	10,7590	3.355,88	3.355,88
	9	231,26	*231,20*	$P_{i,\,min} \le P_i \le P_{i,max}$	11,2335	11,2323	2.646,62	2.645,70
	10	300,00	300,00	$P_i = P_{i,max}$	10,7590	10,7590	3.355,88	3.355,88
	11	200,73	*200,60*	$P_{i,\,min} \le P_i \le P_{i,max}$	11,2335	11,2323	2.399,77	2.398,37
	12	300,00	300,00	$P_i = P_{i,max}$	11,1793	11,1793	3.359,67	3.359,67
	13	300,00	300,00	$P_i = P_{i,max}$	11,1886	10,4706	3.422,87	3.315,18
	14	309,82	*309,50*	$P_{i,\,min} \le P_i \le P_{i,max}$	11,2335	11,2323	3.505,64	3.502,58
	15	268,43	*268,30*	$P_{i,\,min} \le P_i \le P_{i,max}$	11,2335	11,2323	3.043,57	3.041,77
	16	309,79	*309,50*	$P_{i,\,min} \le P_i \le P_{i,max}$	11,2335	11,2323	3.505,39	3.502,58
Total Generated MW		3000,91	*3.000,00*					
λ_λ (€/MWh)					*11,2335*	*12,0185*		
Total Minimal Operational Costs **(€/h)**							48.860,86	*48.742,93*

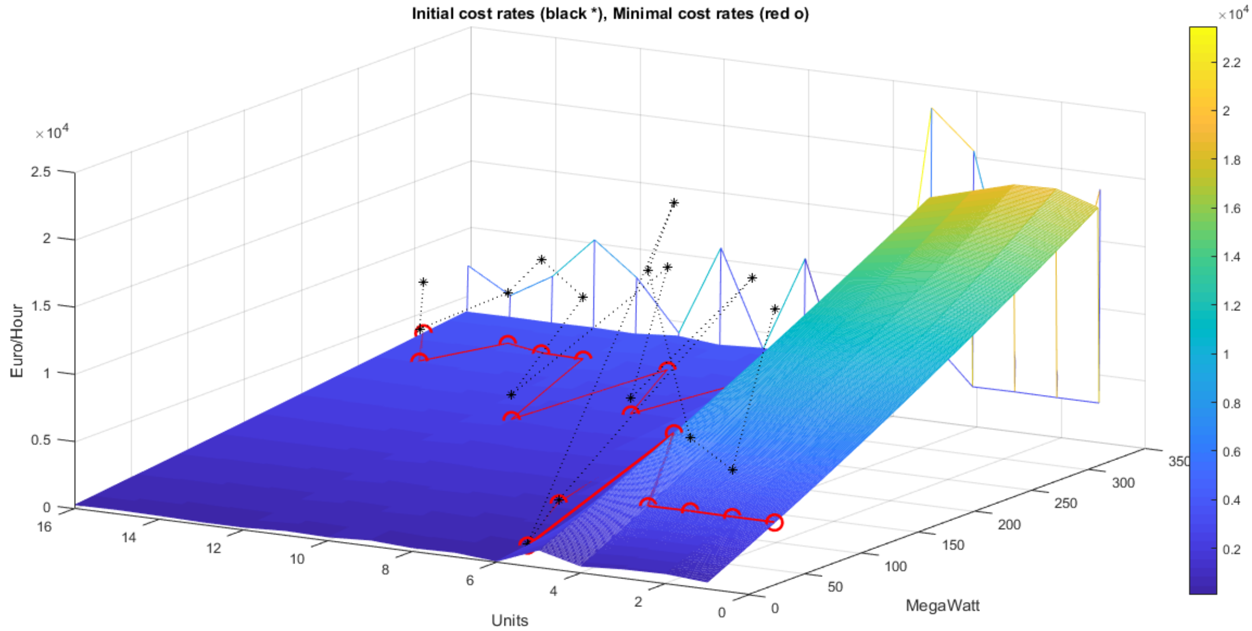

Figure 6. Optimal trajectory for cost rates of 16 generating units: the initial costs-unconstrained optimal (black *****), the minimal costs-constrained optimal (red **O**). On the back surface y-z are plotted the projections of initial and minimal costs (from Fig. 5)

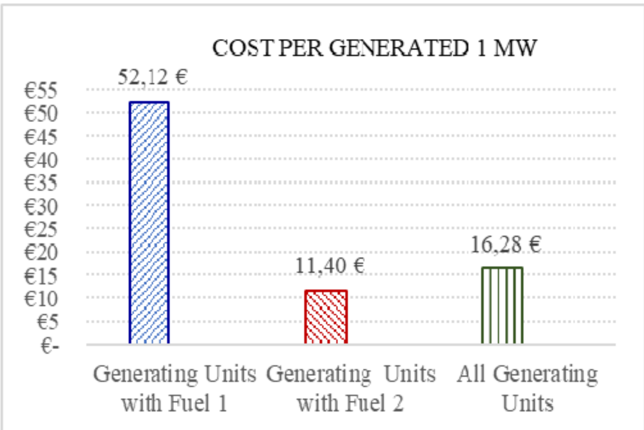

Figure 7. Costs per generated 1 MW from different fuels

TABLE II. MINIMAL COSTS OF GENERATED MW FROM DIFFERENT FUELS

Mean Minimal Cost of generating Units with Fuel 1 (€/h)	3.752,47
Minimal Cost of Generated 1 MW by Generating Units with Fuel 1 (€/h)	52,12
Mean Minimal Cost of generating Units with Fuel 2 (€/h)	2736,23
Minimal Cost of Generated 1 MW by Generating Units with Fuel 2 (€/h)	11,40
Mean Minimal Cost of all generating Units (with all fuels) (€/h)	3.053,80
Minimal Cost of Generated 1 MW by all Generating Units (€/h)	16,28

We see that the cost of 1 MW generated by all generating units with all fuels at €16,28, is higher than the cost of 1 MW generated from Units with Fuel 2, (lignite) at €11,40, and is lower than the cost of 1 MW generated by Units with Fuel 1, (heavy oil) at €52,12.

Based on to the results from Table II, we conclude that the costs of different kind of fuels influence the final operational cost of generated electric power, Fig 7.

From the above discussion results that, the operational cost of lignite-based production of electrical energy is lower than the operational cost for production of electric energy from other fuels, and, that the combination of lignite with other fuels diminishes the final minimal cost per 1 MW.

VIII. STUDY CASE 2. FOURTEEN GENERATING UNITS FROM IEEE 118 BUS SYSTEM

In Study Case 2 we used data from reference [50] and made the comparison with the results obtained using software EGM_ED. The second test system is the IEEE 118-bus that contains 118 buses and 14 thermal generators. The generating unit's data of the IEEE 118-bus were retrieved from [50] and are in Table A2 in Appendix A. They have smooth quadratic (convex) fuel cost characteristic functions, and operate with one kind of fuel. The study cases from [50] are at load demands of 950 MW, 1500 MW and 2650 MW, neglecting transmission losses. Our software EGM_ED was tested on the IEEE 118 bus, with the same cost characteristics from Table A2, and operating exactly at the same load demands of 950 MW, 1500 MW and 2650 MW. In Table III are the results computed using software EGM_ED and the corresponding to Economic Dispatch only are retrieved from [50], for the three load demands: 950 MW, 1500 MW, 2650 MW.

Using EGM_ED we found a smaller total minimal cost in all three cases.

TABLE III. RESULTS FOR ED PROBLEM FOR MODIFIED IEEE 118-BUS SYSTEM WITH 14 THERMAL GENERATING UNITS

Algorithm & Software EGM_ED		**Algorithm from [50]**		
Total Generated Power (MW)	**Minimal Operational Costs ($/h)**	**Load Demand (MW)**	**Minimal Costs ($/h)**	**Difference between Minimal Costs EGM_ED and [50] ($/h)**
950,57	4276,35	950	4.407,95	-131,60
1503,20	6160,33	1500	6.183,60	-23,27
2647,35	11199,78	2650	11.315,97	-116,19

IX. CONCLUSION

There is a clear *Added Value* of new knowledge from our new generalized model and software for solving ED problems using matrix mathematics, which consists of the followings:

The EGM_ED algorithm solves first the unconstrained optimal ED problem, and finds the global optimal solution for incremental cost. This global optimal becomes the starting point, with the initial values of incremental cost and generated powers, for solving the constrained optimal ED problem. From the global optimal solution, taking into consideration the boundary conditions and constraints, the EGM_ED algorithm searches for the generating units which will be set to minimum, or to maximum operating points, and the generating units which will operate between limits.

The constrained minimal total operational cost is found, and depends on a specific combination of parameters of all generating units, load demand, power balance, power bound conditions and allowed convergence errors.

The EGM_ED accepts and solves energy systems with big numbers of power generating units, with many boundary operating zones, boundary conditions, and many different fuels and prices. The resolution of incremental costs, and of the allowed errors of generated power influence the number of iterations for convergence of algorithm. However, in all situations, EGM_ED software finds a fast and accurate solution.

The *Educational benefits* are: experiments with study cases from real power plants solved in laboratory attract the increased interest of students. The students receive the complete information, and take into consideration that all ED problems are complicated and need a multidisciplinary knowledge and multicriteria managerial decisions. They learned how to mathematically solve the problem, without using solvers from toolboxes, ready for input data. Graduates improved their professional knowledge and competencies searching for future jobs.

The *Economic Management* of energy systems discipline enriches with topics of mathematics, optimal control, software, and business. Knowledge gained from lessons of energy systems economics, both theory, and experiments in laboratory, extends to basic, and applied research.

Our model, and software EGM_ED can be accessed, and used by engineers, students and practitioners, for many other related applications of ED using modern software.

APPENDIX A

In Table A1 are the data of sixteen thermal units Unit1-Unit16 from the Greek power system, the output powers P_{min} and P_{max}, the heat consumption/hour *H(P)*, and the kind of fuel. Fuel costs are: Fuel 1, heavy full oil cost rate is 25,50 €/Gcal, Fuel 2, Lignite cost rate is 6,00 €/Gcal.

In Table A2 are the data of fourteen thermal units from IEEE 118-bus: output powers P_{min} , P_{max}, and Fuel Cost rates per hour.

TABLE A1. DATA OF 16 THERMAL UNITS

Generating Units		**P_{min} (MW)**	**P_{max} (MW)**	**Heat Consumption/hour** $H(P)=H_{i,2} \cdot P^2+H_{i,1} \cdot P+H_{i,0}$ *(Gcal/h)*
Fuel 1. Heavy Full Oil	*1*	*60*	*150*	$0,0008286 \cdot P^2+1,6678 \cdot P+19,02$
	2	*60*	*150*	$0,0016573 \cdot P^2 +1,4689 \cdot P+28,03$
	3	*60*	*150*	$0,001450 \cdot P^2 +1,5353 \cdot P+ 25,78$
	4	*60*	*160*	$0,000780 \cdot P^2 +1,6868 \cdot P+ 18,24$
	5	*120*	*300*	$0,0005697 \cdot P^2 +1,5682 \cdot P+49,80$
Fuel 2. Lignite	6	28	70	$0,0051020 \cdot P^2 +1,7286 \cdot P+ 12,80$
	7	70	130	$0,0050717 \cdot P^2 +0,9301 \cdot P+ 55,19$
	8	120	300	$0,0002536 \cdot P^2 +1,6410 \cdot P +44,19$
	9	120	300	$0,0012166 \cdot P^2 +1,3095 \cdot P +73,20$
	10	120	300	$0,0002536 \cdot P^2 +1,6410 \cdot P +44,19$
	11	150	300	$0,0007983 \cdot P^2 +1,5518 \cdot P+ 56,32$
	12	150	300	$0,0003354 \cdot P^2 +1,662 \cdot P+ 31,16$
	13	170	300	$0,0002216 \cdot P^2 +1,7318 \cdot P+ 30,99$
	14	170	310	$0,0003985 \cdot P^2 +1,6253 \cdot P+ 42,47$
	15	170	300	$0,0006215 \cdot P^2 +1,5386 \cdot P+ 49,49$
	16	170	310	$0,0003985 \cdot P^2+1,6253 \cdot P+ 42,47$
Total MW		*1.798*	*3.825*	

TABLE A2. DATA OF 14 THERMAL UNITS FROM IEEE 118-BUS, [50]

Generating Units		**P_{min} (MW)**	**P_{max} (MW)**	**Fuel Cost / hour** ***F(P)*** ***($/h)***
Fuel 1. Lignite	*1*	50	300	$0.005 \cdot P^2 + 1,89 \cdot P + 150$
	2	50	300	$0.0055 \cdot P^2 + 2 \cdot P + 115$
	3	50	300	$0.006\ P^2 + 3,50 \cdot P + 40$
	4	50	300	$0,005 \cdot P^2 + 3,15 \cdot P + 122$
	5	50	300	$0,005 \cdot P^2 + 3,05 \cdot P + 125$
	6	50	300	$0,007 \cdot P^2 + 2,75 \cdot P + 70$
	7	50	300	$0,007 \cdot P^2 + 3,45 \cdot P + 70$
	8	50	300	$0,007 \cdot P^2 + 3,45 \cdot P + 70$
	9	50	300	$0,005 \cdot P^2 + 2,45 \cdot P + 130$
	10	50	300	$0,005 \cdot P^2 + 2,45 \cdot P + 130$
	11	50	300	$0,0055 \cdot P^2 + 2,35 \cdot P + 135$
	12	50	300	$0,0045 \cdot P^2 + 1,30 \cdot P + 200$
	13	50	300	$0,007 \cdot P^2 + 3,45 \cdot P + 70$
	14	50	300	$0,006 \cdot P^2 + 3,89 \cdot P + 45$
Total MW		*700*	*4200*	

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